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Structured Additive Regression Models: An R Interface to BayesX

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Abstract

Structured additive regression (STAR) models provide a flexible framework for modeling possible nonlinear effects of covariates: They contain the well established frameworks of generalized linear models and generalized additive models as special cases but also allow a wider class of effects, e.g., for geographical or spatio-temporal data, allowing for specification of complex and realistic models. BayesX is standalone software package providing software for fitting general class of STAR models. Based on a comprehensive open-source regression toolbox written in C++, BayesX uses Bayesian inference for estimating STAR models based on Markov chain Monte Carlo simulation techniques, a mixed model representation of STAR models, or stepwise regression techniques combining penalized least squares estimation with model selection. BayesX not only covers models for responses from univariate exponential families, but also models from less-standard regression situations such as models for multi-categorical responses with either ordered or unordered categories, continuous time survival data, or continuous time multi-state models. This paper presents a new fully interactive R interface to BayesX: the R package R2BayesX. With the new package, STAR models can be conveniently specified using R's formula language (with some extended terms), fitted using the BayesX binary, represented in R with objects of suitable classes, and finally printed/summarized/plotted. This makes BayesX much more accessible to users familiar with R and adds extensive graphics capabilities for visualizing fitted STAR models. Furthermore, R2BayesX complements the already impressive capabilities for semiparametric regression in R by a comprehensive toolbox comprising in particular more complex response types and alternative inferential procedures such as simulation-based Bayesian inference.

Keywords: STAR models, MCMC, REML, stepwise, R.

1. Introduction

The free software **BayesX** (see Brezger, Kneib, and Lang 2005) is a standalone program (current version 2.1, Belitz, Brezger, Kneib, and Lang 2012) comprising powerful tools for Bayesian, mixed-model-based and stepwise inference in complex semiparametric regression models with structured additive predictor. Besides exponential family regression, **BayesX** also supports models for multi-categorical responses, hazard regression for continuous survival times, and continuous time multi-state models. The software is written in C++, utilizing numerically efficient (sparse) matrix architectures.

To facilitate usage of results from **BayesX** in subsequent analyses, specifically in explorations and visualizations of the fitted models, Kneib, Heinzl, Brezger, Bove, and Klein (2014) provide a package for R (R Core Team 2014), also called **BayesX**, that can read and process output files from **BayesX**. However, in this approach the users still have to read their data into **BayesX**, fit the models of interest and obtain the corresponding output files. To alleviate this task, we introduce a new R package **R2BayesX** that provides a fully interactive R interface to **BayesX** that has the usual R modeling "look & feel" and obviates the tedious exercise of manually exporting data and fitting models in **BayesX**. Within the new package, users are now provided with the typical R modeling workflow namely:

- Specification and estimation of STAR models using bayesx(formula, data, ...) (which internally calls BayesX and reads its results).
- Methods and extractor functions for fitted "bayesx" model objects, e.g., producing highlevel graphics of estimated effects, model diagnostic plots, summary statistics etc.

In addition, users can leverage the underlying infrastructure, i.e.:

- Run already existing BayesX input program files from R via run.bayesx().
- Automatically import BayesX output files into R via read.bayesx.output().

The formula interface of the bayesx() function uses the special model term constructor function sx() for structured predictors (such as smooth, spatial, or random effects). Internally this leverages smooth term functionality from the mgcv package (Wood 2014, 2006), facilitating a consistent way to translate R syntax into BayesX-interpretable commands.

The functionality is made available in package R2BayesX, available from the Comprehensive R Archive Network (CRAN) at http://CRAN.R-project.org/package=R2BayesX. It depends on the companion package BayesXsrc (also available from CRAN, see Adler, Kneib, Lang, Umlauf, and Zeileis 2014) that ships the BayesX C++ sources along with flexible Makefiles so that upon installation of the R package a suitable BayesX binary is produced on all platforms.

The remainder of this paper is as follows. Section 2 gives a first motivating example of an R session applying R2BayesX to a dataset on childhood malnutrition in Zambia. Subsequently, Sections 3–5 introduce the underlying methodological and computational building blocks before Section 6 returns to more advanced illustrations of using R2BayesX in practice (for the childhood malnutrition data and a dataset on forest health in Germany). More specifically, Section 3 briefly discusses the methodological background of structured additive regression models, Section 4 covers the interface design and implementation details, and Section 5 describes the user interface. These three sections are written in a modular style so that

they can be easily skipped (or read at a later time) by readers that are less interested in the underlying methodological/computational details (especially the interface design in Section 4) and more interested in applying **R2BayesX** in practice. Section 7 concludes the paper and further technical details are provided in Appendices A, B, and C.

2. Motivating example

To give an introductory example of the various features of the interface, we estimate a Bayesian geoadditive regression model for the childhood malnutrition dataset in Zambia (see Kandala, Lang, Klasen, and Fahrmeir 2001 and also Section 6.1) using Markov chain Monte Carlo (MCMC) simulation.

The data consists of 4847 observations including 8 variables, both continuous and categorical. In this analysis, the main interest is assessment of the determinats of stunting (stunting), represented by anthropometric indicators of newborn children. Covariates include the age of the children (agechild), the body mass index (BMI) of the mother (mbmi) and the district the children live in (district). The model is given by

```
\mathtt{stunting}_i = \gamma_0 + f_1(\mathtt{agechild}_i) + f_2(\mathtt{mbmi}_i) + f_{spat}(\mathtt{district}_i) + \varepsilon_i, \qquad \varepsilon_i \sim N(0, \sigma^2),
```

where the functions f_1 and f_2 of continuous covariates agechild and mbmi have possible nonlinear effects on stunting and are modeled nonparametrically using P(enalized)-splines. Here, the spatially correlated effect f_{spat} of locational covariate district is modeled using kriging based on centroid coordinates (geokriging) of the districts in Zambia. To estimate the model with **BayesX** from R, the data together with a map of the districts in Zambia (see Section 5.2 for details of the map format) is loaded with

```
R> data("ZambiaNutrition", "ZambiaBnd", package = "R2BayesX")
```

Then, the model can be fitted to a suitable formula with the main model-fitting function bayesx()

```
R> b <- bayesx(stunting ~ sx(agechild) + sx(mbmi) +
+ sx(district, bs = "gk", map = ZambiaBnd),
+ family = "gaussian", method = "MCMC", data = ZambiaNutrition)</pre>
```

The model summary is displayed by calling

```
R> summary(b)
```

Call:

```
bayesx(formula = stunting ~ sx(agechild) + sx(mbmi) + sx(district,
    bs = "gk", map = ZambiaBnd), data = ZambiaNutrition, family = "gaussian",
    method = "MCMC")
```

Fixed effects estimation results:

Parametric coefficients:

```
Mean Sd 2.5% 50% 97.5% (Intercept) 0.0310 0.0427 -0.0507 0.0323 0.1128
```

Smooth terms variances:

```
Mean Sd 2.5% 50% 97.5% Min Max sx(agechild) 0.0066 0.0063 0.0012 0.0045 0.0244 0.0004 0.0657 sx(district) 0.0451 0.0185 0.0214 0.0413 0.0952 0.0148 0.1511 sx(mbmi) 0.0022 0.0034 0.0003 0.0012 0.0098 0.0002 0.0549
```

Scale estimate:

Mean Sd 2.5% 50% 97.5% Sigma2 0.8197 0.0172 0.7870 0.8194 0.8555

```
N = 4847 burnin = 2000 DIC = 4876.806 pd = 32.308 method = MCMC family = gaussian iterations = 12000 step = 10
```

A plot of the estimated effect for covariate mbmi may then be produced by typing

```
R> plot(b, term = "sx(mbmi)")
```

and for covariate agechild including partial residuals by

```
R> plot(b, term = "sx(agechild)", residuals = TRUE)
```

The estimated effect of the correlated spatial effect of the districts in Zambia may e.g., be visualized using a map effect plot generated by

```
R> plot(b, term = "sx(district)", map = ZambiaBnd)
```

The plots are shown in Figure 1, depicting the centered additive effects (i.e., each of the additive effects is zero on average). The map effect plot indicates pronounced stunting (i.e., low values of the response) in the northern parts of Zambia. Furthermore, stunting effects are lower (i.e., the response is higher) for children younger than 20 months of age, while the agechild effect is almost constant for ages above 20 months. Finally, the response increases almost linearly with increasing mother's BMI. In comparison, the effects of mbmi and the spatial effect seem to have a quite similar influence in absolute magnitude (indicated by the ranges of the respective axes), while the strongest driver of stunting appears to be covariate agechild. Extended analyses of the data are discussed in Sections 6.1 and 6.3.

3. STAR models

The STAR model class supported by **R2BayesX** is based on the framework of Bayesian generalized linear models (GLMs, see e.g., McCullagh and Nelder 1989 and Fahrmeir and Tutz 2001). GLMs assume that, given covariates \mathbf{x} and unknown parameters $\boldsymbol{\gamma}$, the distribution of the response variable y belongs to an exponential family with mean $\mu = E(y|\mathbf{x},\boldsymbol{\gamma})$ linked to a linear predictor η by

$$\mu = h^{-1}(\eta), \qquad \eta = \mathbf{x}^{\mathsf{T}} \boldsymbol{\gamma},$$

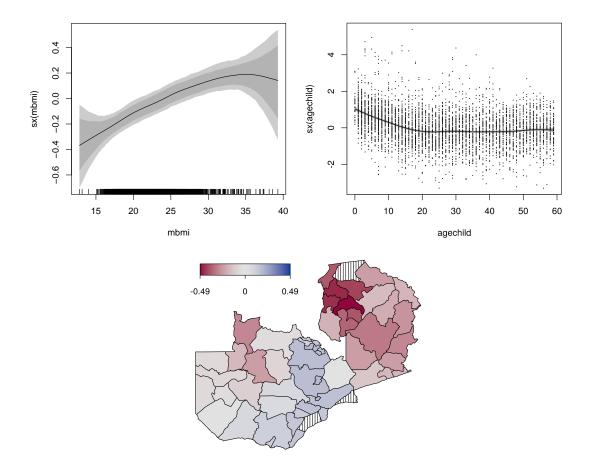


Figure 1: Visualization examples: Estimated effect for covariate mbmi (black line) together with 95% and 80% credible intervals (upper left panel). The upper right panel shows the estimated effect of agechild. The lower panel illustrates visualization of the estimated spatial effect for covariate district using a map effect plot (regions with vertical lines represent areas with no observations).

where h is a known link function and γ are unknown regression coefficients. In STAR models (Fahrmeir, Kneib, and Lang 2004; Brezger and Lang 2006), the linear predictor is replaced by a more general and flexible, structured additive predictor

$$\eta = f_1(\mathbf{z}) + \ldots + f_p(\mathbf{z}) + \mathbf{x}^{\top} \boldsymbol{\gamma}, \tag{1}$$

with $\mu = E(y|\mathbf{x}, \mathbf{z}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ and \mathbf{z} represents a generic vector of all nonlinear modeled covariates. The vector $\boldsymbol{\theta}$ comprises all parameters of the functions f_1, \ldots, f_p . The functions f_j are possibly smooth functions encompassing various types of effects, e.g.:

- Nonlinear effects of continuous covariates: $f_i(\mathbf{z}) = f(z_1)$.
- Two-dimensional surfaces: $f_j(\mathbf{z}) = f(z_1, z_2)$.
- Spatially correlated effects: $f_j(\mathbf{z}) = f_{spat}(z_s)$.
- Varying coefficients: $f_j(\mathbf{z}) = z_1 f(z_2)$.

- Spatially varying effects: $f_j(\mathbf{z}) = z_1 f_{spat}(z_s)$ or $f_j(\mathbf{z}) = z_1 f(z_2, z_3)$.
- Random intercepts with cluster index c: $f_i(\mathbf{z}) = \beta_c$.
- Random slopes with cluster index c: $f_i(\mathbf{z}) = z_1 \beta_c$.

STAR models cover a number of well known model classes as special cases, including generalized additive models (GAM, Hastie and Tibshirani 1990), generalized additive mixed models (GAMM, Lin and Zhang 1999), geoadditive models (Kamman and Wand 2003), varying coefficient models (Hastie and Tibshirani 1993), and geographically weighted regression (Fotheringham, Brunsdon, and Charlton 2002).

The unified representation of a STAR predictor arises from the fact that all functions f_j in (1) may be specified by a basis function approach, where the vector of function evaluations $\mathbf{f}_j = (f_j(\mathbf{z}_1), \dots, f_j(\mathbf{z}_n))^{\top}$ of the $i = 1, \dots, n$ observations can be written in matrix notation

$$\mathbf{f}_j = \mathbf{Z}_j \boldsymbol{\beta}_j,$$

where the design matrix \mathbf{Z}_j depends on the specific term structure chosen for f_j and $\boldsymbol{\beta}_j$ are unknown regression coefficients to be estimated. Hence, the predictor (1) may be rewritten as

$$\boldsymbol{\eta} = \mathbf{Z}_1 \boldsymbol{\beta}_1 + \ldots + \mathbf{Z}_p \boldsymbol{\beta}_p + \mathbf{X} \boldsymbol{\gamma},$$

where **X** corresponds to the usual design matrix for the linear effects.

To ensure particular functional forms, prior distributions are assigned to the regression coefficients. The general form of the prior for β_j is

$$p(\boldsymbol{\beta}_j | \tau_j^2) \propto \exp\left(-\frac{1}{2\tau_j^2} \boldsymbol{\beta_j}^{\top} \mathbf{K}_j \boldsymbol{\beta}_j\right),$$

where \mathbf{K}_j is a quadratic penalty matrix that shrinks parameters towards zero or penalizes too abrupt jumps between neighboring parameters. In most cases \mathbf{K}_j will be rank deficient and the prior for $\boldsymbol{\beta}_j$ is partially improper.

The variance parameter τ_j^2 is equivalent to the inverse smoothing parameter in a frequentist approach and controls the trade off between flexibility and smoothness. For full Bayesian inference, weakly informative inverse Gamma hyperpriors $\tau_j^2 \sim IG(a_j,b_j)$ are assigned to τ_j^2 , with $a_j = b_j = 0.001$ as a standard option. Small values for a_j and b_j correspond to an approximate uniform distribution for $\log \tau_j^2$. For empirical Bayes inference, τ_j^2 is considered an unknown constant which is determined via restricted maximum likelihood (REML).

In **BayesX**, estimation of regression parameters is based on three inferential concepts:

1. Full Bayesian inference via MCMC

A fully Bayesian interpretation of STAR models is obtained by specifying prior distributions for all unknown parameters. Estimation is carried out using MCMC simulation techniques. **BayesX** provides numerically efficient implementations of MCMC schemes for structured additive regression models. Suitable proposal densities have been developed to obtain rapidly mixing, well-behaved sampling schemes without the need for manual tuning (Brezger and Lang 2006).

2. Inference via a mixed model representation

Another concept used for estimation is based on mixed model methodology. The general idea is to take advantage of the close connection between penalty concepts and corresponding random effects distributions. The smoothing variances of the priors then transform to variance components in the random effects (mixed) model. While regression coefficients are estimated based on penalized likelihood, restricted maximum likelihood or marginal likelihood estimation forms the basis for the determination of smoothing parameters. From a Bayesian perspective, this yields empirical Bayes/posterior mode estimates for the STAR models. However, estimates can also merely be interpreted as penalized likelihood estimates from a frequentist perspective (Fahrmeir et al. 2004).

3. Penalized likelihood including variable selection

As a third alternative **BayesX** provides a penalized least squares (or penalized likelihood) approach for estimating STAR models. In addition, a powerful variable and model selection tool is included. Model choice and estimation of the parameters is done simultaneously. The algorithms are able to

- decide whether a particular covariate enters the model,
- decide whether a continuous covariate enters the model linearly or nonlinearly,
- decide whether a spatial effect enters the model,
- decide whether a unit- or cluster-specific heterogeneity effect enters the model,
- select complex interaction effects (two dimensional surfaces, varying coefficient terms),
- select the degree of smoothness of nonlinear covariate, spatial or cluster specific heterogeneity effects.

Inference is based on penalized likelihood in combination with fast algorithms for selecting relevant covariates and model terms. Different models are compared via various goodness of fit criteria, e.g., Akaike or Bayes information criterion (AIC or BIC), generalized cross-validation (GCV), or 5- or 10-fold cross-validation (Belitz and Lang 2008).

A thorough introduction into the regression models supported by the program is also provided in the **BayesX** methodology manual (Belitz *et al.* 2012). An in depth discussion using a number of empirical examples is also provided in Fahrmeir, Kneib, Lang, and Marx (2013). Further details on special cases of STAR models are also provided in Fahrmeir *et al.* (2004).

Software packages which have a similar Bayesian scope are the R package INLA (Rue, Martino, and Chopin 2009; Lindgren and Rue 2015), BUGS/WinBUGS (Lunn, Spiegelhalter, Thomas, and Best 2009; Lunn, Thomas, Best, and Spiegelhalter 2000), JAGS (Plummer 2003) and Stan (Stan Development Team 2014). While the former package is relatively similar in its workflow compared to R2BayesX and works with hierarchical Gaussian Markov random fields (GMRF), Bayesian inference is not based on MCMC sampling but on integrated nested Laplace approximation (INLA). This has the advantage of avoiding questions concerning mixing and convergence but also requires quite advanced mathematical and numerical tools for implementation. Moreover, complex hierarchical prior structures (such as the Bayesian lasso) or models with a large number of hyperparameters are more difficult to handle in the INLA framework. In summary, R2BayesX and INLA address special model classes using optimized

algorithms, the latter packages apply Bayesian inference using Gibbs sampling, where the user can in principle program a number of very complex models, also the ones covered by **R2BayesX**, but due to their flexible MCMC samplers and the data management/efficiency, STAR models usually run very long and show inferior sampling properties. A detailed comparison of **BayesX** with other software packages including **WinBUGS** is provided in Brezger et al. (2005, Section 3). See also Section 4.2 for more R packages, that deal with GLM and GAM models, mainly from a frequentist perspective.

4. Implementation of the R interface to BayesX

The design of the interface attempts to address the following major issues: First, the interface functions should follow R's conventions for regression model fitting functions so that they are easy to employ for R users. Second, the functions and methods for representing fitted model objects should reflect **BayesX** models to enhance their usability.

This section takes a developer's perspective and discusses the design choices in **R2BayesX** and the technical issues involved while the subsequent Section 5 takes a user's perspective, providing an introduction on how to employ the **R2BayesX** for STAR modeling.

4.1. Interface approach

The first challenge in establishing a communication between R and BayesX is the question which interface to use. As BayesX is written in C++, one might expect that .C() or .Call() could be an option. However, as BayesX was designed as a standalone software it does not offer an application programming interface (API) and restructuring the mature and complex BayesX C++ code to obtain an API at this point is not straightforward. Hence, R2BayesX adopts the simpler approach of writing the data out from R, calling the BayesX binary with a suitable program file, and then collecting all output files and representing them in suitable R objects. This is straightforward and the additional computation effort (as compared to a direct call) is rather modest compared to time needed for carrying out the estimation of STAR models within BayesX.

Thus, for the interface adopted by R2BayesX a binary installation of BayesX is required. To make this easily available to R users in a standardized way, the BayesX C++ sources are encapsulated in R package BayesXsrc along with Makefiles for GNU/BSD and MinGW platforms that conform with R build shells. Consequently, upon installation of the BayesXsrc package, the binary BayesX (or BayesX.exe on Windows) is created in the installed package. Package BayesXsrc is also available from CRAN at http://CRAN.R-project.org/package=BayesXsrc and some of its implementation details are discussed in Appendix A.

4.2. Model specification

The second challenge for the interface package **R2BayesX** is to employ an objects and methods interface that reflects the workflow typically adapted by R packages for fitting GAMs and related models. CRAN packages that implement such models include the following prominent ones: One of the first implementations of GAMs in R is the **gam** package (Hastie and Tibshirani 1990; Hastie 2013). The package is supporting local regression and smoothing splines in combination with a backfitting algorithm and is actually a version of the S-PLUS routines for

GAMs. The probably best-known and also recommended package is **mgcv** (Wood 2006, 2011, 2014), which provides fast and stable algorithms for estimating GAMs based on GCV, REML and others. Vector generalized additive models (VGAMs, Yee and Wild 1996) for categorical responses are covered by package **VGAM** (Yee 2010). Another comprehensive toolbox for GAMs, accounting for responses that do not necessarily follow the exponential family and may exhibit heterogeneity, is the **gamlss** suite of packages (Rigby and Stasinopoulos 2005; Stasinopoulos and Rigby 2007). A package based on mixed model technologies is **SemiPar** (Ruppert, Wand, and Carrol 2003; Wand 2013) and, building on top of this, the **Adapt-Fit** package for adaptive splines (Krivobokova 2012). The package **spikeSlabGAM** applies Bayesian variable selection, model choice and regularization for GAMMs (Scheipl 2011).

Most of these packages follow the common R paradigm of specifying regression models conveniently using its formula language (Chambers and Hastie 1992). However, the above-mentioned packages employ somewhat different approaches for representing smooth/special terms for GAMs in formulas and the subsequent building of model frames. A popular approach, though, is to use a model term constructor function "s", as used in packages gam, mgcv, and VGAM. As the implementation details are somewhat different across these packages, loading packages simultaneously may lead to conflicts. Therefore, R2BayesX follows the approach of the recommended package mgcv where s() does not evaluate design or penalty matrices, but simply returns a smooth term definition object of class "xx.smooth.spec", where "xx" may be specified by the user. To set up a model with a user-defined smooth term, a method for the S3 generic function smooth.construct() needs to be supplied, that returns a design matrix etc. Since implementation of additional model terms is also a concern for R2BayesX and function s() is a very lean solution, we adopt its functionality and provide methods for a new generic function bayesx.construct(), that returns the required command for a particular smooth term in BayesX.

Given an R model formula, the specified terms are translated one after another and finally merged into a complete program which may be sent to **BayesX**. Note, however, that due to different estimation methods in **mgcv** and **BayesX** the default recommendations for specification of a given basis (e.g., P-splines) differ. To account for this a new smooth term constructor sx() is provided that is recommended as the principal user interface in **R2BayesX** and described in Section 5.2 in detail. For example, the defaults for a P-spline in **BayesX** are

```
R > bayesx.construct(sx(x, bs = "ps"))
```

[1] "x(psplinerw2,nrknots=20,degree=3)"

Internally, sx() simply calls mgcv's s() to set up the smooth term but it chooses the defaults in accordance with BayesX. A detailed account how arguments are mapped between sx() and s() is provided in Appendix C.

4.3. Under the hood

The main user interface of **R2BayesX** is the function bayesx() (presented in detail in Section 5.1). Internally, this function employs the helper functions parse.bayesx.input(), write.bayesx.input(), run.bayesx(), and read.bayesx.output() in the following work sequence: First, a program file is generated by applying function parse.bayesx.input() to the R input parameters, including the model formula, data, etc. The returned object

is then further processed with function write.bayesx.input(), utilizing the methods described above, as well as setting up the necessary temporary directories and data files to be used with BayesX. Afterwards, function run.bayesx() (provided in BayesXsrc) executes the program through a call to function system(). The output files returned by the binary are imported into R using function read.bayesx.output(). Using these helper functions it is also possible to run and read already existing BayesX program and output files, see Appendix D and the R2BayesX manuals for a detailed description. The object returned by function read.bayesx.output() is a list of class "bayesx", for which a set of base R functions and methods described in Table 3, amongst others, is available. The returned fitted model term objects also have suitable classes along with corresponding plotting methods. Particular effort has been given on the development of easy-to-use map effect plots using color legends (by default employing HCL-based palettes, Zeileis, Hornik, and Murrell 2009, from the colorspace package, Ihaka, Murrell, Hornik, Fisher, and Zeileis 2013). See also Section 5 for more details and Section 6 for some practical applications.

5. User interface

5.1. Calling BayesX from R

The main model-fitting function in the package $\mathbf{R2BayesX}$ is called $\mathtt{bayesx}()$. The arguments of $\mathtt{bayesx}()$ are

```
bayesx(formula, data, weights = NULL, subset = NULL, offset = NULL,
  na.action = NULL, contrasts = NULL,
  family = "gaussian", method = "MCMC", control = bayesx.control(...),
  chains = NULL, cores = NULL, ...)
```

where the first two lines basically represent the standard model frame specifications (see Chambers and Hastie 1992) and the third line collects the arguments specific to **BayesX**.

The data processing is carried out "as usual" as in lm() or glm() with the following additions: (1) The data can not only be provided as a "data.frame" but it is also possible to provide a character string with a path to a dataset stored on disc, which can be leveraged to avoid reading very large data files into R just to write them out again for BayesX. An example is given in Appendix D. (2) Additional contrast specifications for factor variables can be passed to argument contrasts. Using factors, we recommend deviation or effect coding (see function contr.sum()) rather than the usual dummy coding of factors as it typically improves convergence of estimation algorithms used in BayesX.

The BayesX-specific arguments comprise specification of the response distribution family, the estimation method and further control parameters collected in bayesx.control(). The default response distribution is family = "gaussian". Note that "family" objects (in the glm() sense) are currently not supported by BayesX. The inferential concepts that can be used as the estimation method comprise: "MCMC" for Markov chain Monte Carlo simulation, "REML" for mixed-model-based estimation using restricted maximum likelihood/marginal likelihood, and "STEP" for penalized likelihood including model selection. An overview of all available distributions for the different methods is given in Table 1.

family	Response distribution	Link	method
"binomial"	binomial	logit	"MCMC" "REML" "STEP"
"binomialprobit"	binomial	probit	"MCMC" "REML" "STEP"
"gamma"	gamma	\log	"MCMC" "REML" "STEP"
"gaussian"	Gaussian	identity	"MCMC" "REML" "STEP"
"multinomial"	unordered multinomial	logit	"MCMC" "REML" "STEP"
"poisson"	Poisson	\log	"MCMC" "REML" "STEP"
"cox"	continuous-time sur-		"MCMC" "REML"
	vival data		
"cumprobit"	cumulative threshold	probit	"MCMC" "REML"
"multistate"	continuous-time multi-		"MCMC" "REML"
	state data		
"binomialcomploglog"	binomial	compl.	"REML"
		log-log	
"cumlogit"	cumulative multinomial	logit	"REML"
"multinomialcatsp"	unordered multinomial	logit	"REML"
	(with category-specific		
	covariates)		
"multinomialprobit"	unordered multinomial	probit	"MCMC"
"seqlogit"	sequential multinomial	logit	"REML"
"seqprobit"	sequential multinomial	probit	"REML"

Table 1: Distributions implemented for methods "MCMC", "REML" and "STEP".

The user can additionally run "MCMC" models on multiple chains and cores, e.g., to check convergence of the samples (for an example see Section 6.1). While the latter is not supported on Windows system, multiple chains can be started on all platforms.

The last argument specifies several parameters controlling the processing of the **BayesX** binary that are arranged by function bayesx.control(). Note that all additional controlling arguments are automatically parsed within function bayesx() using the dot dot dot argument "...", which is sent to bayesx.control(). The most important parameters for the different methods are listed in Table 2.

The returned fitted model object is a list of class "bayesx", which is supported by several standard methods and extractor functions, such as plot() and summary(). For models estimated using method "REML", function summary() generates summary statistics similar to objects returned from the main model fitting function gam() of the mgcv package. For "MCMC" estimated models, the mean, standard deviation and quantiles of parameter samples are provided. Using "STEP", the parametric part of the summary statistics is represented like "MCMC", i.e., if computed, the confidence bands are based on an MCMC algorithm subsequent to the model selection, while the remaining summary is similar to "REML". The implemented S3 methods for plotting fitted term objects are quite flexible, i.e., depending on the term structure, the generic function plot() calls one of the following functions: for 2d plots function plot2d() or plotblock() (for factors, unit- or cluster specific plots, draws a block for every estimated parameter including mean and credible intervals), for perspective or image and contour plots function plot3d(), map effects plots are produced by function plotmap(), with or without colorlegends drawn by function colorlegend(), amongst others. See Appendix B

method	Parameter	Description
"MCMC"	iterations burnin	Integer number of iterations for the sampler, default: 12000. Integer burn-in period of the sampler, default: 2000.
	step	Integer, defines the thinning parameter for MCMC simulation. E.g., step = 50 means, that only every 50th sampled parameter will be stored and used to compute characteristics of the posterior distribution as means, standard deviations or quantiles, default: 10.
"REML"	eps	Numeric, defines the termination criterion of the estimation process. If both the relative changes in the regression coefficients and the variance parameters are less than eps, the estimation process is assumed to have converged, default: 0.00001.
	maxit	Integer, defines the maximum number of iterations to be used in estimation. Since the estimation process will not necessarily converge, it may be useful to define an upper bound for the number of iterations.
"STEP"	algorithm	Character, specifies the selection algorithm. Possible values are "cdescent1" (adaptive algorithms see Section 6.3 in Belitz et al. 2012), "cdescent2" (adaptive algorithms 1 and 2 with backfitting, see remarks 1 and 2 of Section 3 in Belitz and Lang 2008), "cdescent3" (search according to "cdescent1" followed by "cdescent2" using the selected model in the first step as the start model) and "stepwise" (stepwise algorithm implemented in the gam function of S-PLUS, see Chambers and Hastie 1992), default: "cdescent1".
	criterion	Character, specifies the goodness of fit criterion, possible criterions are: "MSEP" (divides the data randomly into a test-and validation dataset. The test dataset is used to estimate the models and the validation dataset is used to estimate the mean squared prediction error which serves as the goodness of fit criterion to compare different models), "GCV" (generalized cross-validation based on deviance residuals), "GCVrss" (GCV based on residual sum of squares), see e.g., Wood (2006), "AIC" (Akaike information criterion), "AIC_imp" (improved AIC with bias correction for regression models), see e.g., Burnham and Anderson (1998), "BIC" (Bayesian information criterion) "CV5" (5-fold cross validation) "CV10" (10-fold CV), see e.g., Hastie, Tibshirani, and Friedman (2009), and "AUC" (area under the ROC curve, binary response only), default: "AIC_imp".
	startmodel	Character, defines the start model for variable selection. Options are "linear" (model with degrees of freedom equal to one for model terms), "empty" (empty model containing only an intercept), "full" (most complex possible model) and "userdefined" (user-specified model), default: "linear".

Table 2: Most important controlling parameters for the different methods using function bayesx(). See ?bayesx.control for more details.

Function	Description
print()	Simple printed display of the initial call and some additional infor-
_	mation of the fitted model.
<pre>summary()</pre>	Return an object of class "summary.bayesx" containing the relevant
	summary statistics (which has a print() method).
coef()	Extract coefficients of the linearly modeled terms.
<pre>confint()</pre>	Compute confidence intervals of linear modeled terms if method =
	"REML", for "MCMC" the quantiles of the coefficient samples according
	to a specified probability level are computed.
cprob()	Extract contour probabilities of a particular P-spline term, only
	meaningful if method = "MCMC" and argument contourprob is speci-
	fied as an additional argument in the term constructor function sx().
	E.g., in the introductory example, contour probabilities for mbmi are
	estimated with sx(mbmi, bs = "ps", contourprob = 4) (see also
	Section 5.2).
fitted()	Fitted values of either the mean and linear predictor, or a selected
	model term.
residuals()	Extract model or partial residuals for a selected term.
samples()	Extract samples of parameters from MCMC simulation.
<pre>bayesx_logfile()</pre>	Extract the internal BayesX log file.
<pre>bayesx_prgfile()</pre>	Extract the BayesX program file.
bayesx_runtime()	Extract the overall runtime of the BayesX binary.
terms()	Extract terms of model components.
model.frame()	Extract/generate the model frame.
logLik()	Extract fitted log-likelihood, only if method = "REML".
plot()	Either model diagnostic plots or effect plots of particular terms.
<pre>getscript()</pre>	Generate an R script for term effect, diagnostic plots and model sum-
	mary statistics.
AIC(), BIC(),	Compute information criteria, availability is dependent on the method
DIC(), GCV()	used.

Table 3: Functions and methods for objects of class "bayesx". More details are provided in the manual pages.

for an overview of the most important arguments for the plotting functions. For MCMC post-estimation diagnosis, besides the implemented trace and autocorrelation plots, samples of the parameters may also be extracted using function samples(). The sampling paths are provided as a data frame, and hence may easily be converted to objects of class "mcmc" using the coda package (Plummer, Best, Cowles, and Vines 2006) for further analysis (see also Section 6.1). In addition, an R script for the estimated model, including function calls for saving, loading, plotting of term effects and diagnostic plots, may be generated using function getscript(). The produced R script may be useful for less experienced users of the package to get a quick overview of post-estimation commands. Moreover, the script facilitates the final preparation of plots and diagnostics to be included in publications. In some situations it may be useful to inspect the log file generated by the BayesX binary. The file can either be viewed directly during fitting process when setting verbose = TRUE, or it can be extracted from the

fitted model object using function bayesx_logfile(). A list of all available functions and methods of package R2BayesX can be found in Table 3.

5.2. Available additive terms

In package R2BayesX, the main constructor function for specifying additive terms in STAR formulas is called sx(). The function is basically an interface to the term constructor function s() of package mgcv but assures defaults appropriate for working with BayesX, see also Section 4. The arguments of function sx() are

```
sx(x, z = NULL, bs = "ps", by = NA, ...)
```

where x represents the covariate that is used for univariate terms and z is used additionally for bivariate model terms. Argument bs chooses the basis/type of the term, see Table 4 for possible options of bs (and note that some terms have equivalent short and long specifications, e.g., bs = "ps" or bs = "psplinerw2"). Argument by can be a numeric or a factor variable to estimate varying coefficient terms, where the effect of the variable provided to by varies over the range of the covariate(s) of this term. Finally, the "..." argument is used to set term-specific control parameters or additional geographical information.

For example to modify the degree and the inner knots for the P-spline term sx(mbmi) from Section 2, sx(mbmi, degree = 2, knots = 10) could be used. Information about all possible extra arguments for a particular term basis/type can be looked up using function bayesx.term.options(), e.g., possible options for P-splines using "MCMC" are shown by

```
R> bayesx.term.options(bs = "ps", method = "MCMC")
Available options for 'bs = "ps"':
    degree: the degree of the B-spline basis functions.
        Default: integer, 'degree = 3'.
    knots: number of inner knots.
        Default: integer, 'knots = 20'.
...
```

For simplicity, only the first two options are shown here. Note that all default specifications, e.g., the number of equally spaced knots for P-splines, have been thoroughly tested and should usually be well-suited for common regression problems (see also Lang and Brezger (2004) and Brezger and Lang (2006) for a detailed discussion). However, in some situations it might be useful to evaluate the sensitivity of the results when changing certain parameters (such as number of knots, degree of the spline, hyperpriors, etc.), e.g., when modeling highly oscillating functions 20 knots may not be sufficient to capture the overall curvature.

For fitting geoadditive models utilizing spatial information – i.e., by computing suitable neighborhood penalty matrices for terms using Markov random field (MRF) priors, or by calculating the centroids of particular regions for geosplines and geokriging terms – an argument

bs	Description
"rw1", "rw2"	Zero degree P-splines: Defines a zero degree P-spline with first or second order difference penalty. A zero degree P-spline typically estimates for every distinct covariate value in the dataset a separate parameter. Usually there is no reason to prefer zero degree P-splines over higher order P-splines. An exception are ordinal covariates or continuous covariates with only a small number of different values. For ordinal covariates higher order P-splines are not meaningful while zero degree P-splines might be an alternative to modeling nonlinear relationships via a dummy approach with completely unrestricted regression parameters.
"season"	Seasonal effect of a time scale.
"ps",	P-spline with first or second order difference penalty.
"psplinerw1",	
"psplinerw2"	
"te",	Defines a two-dimensional P-spline based on the tensor product of
"pspline2dimrw1"	one-dimensional P-splines with a two-dimensional first order random
	walk penalty for the parameters of the spline.
"kr", "kriging"	Kriging with stationary Gaussian random fields.
"gk",	Geokriging with stationary Gaussian random fields (Fahrmeir et al.
"geokriging"	2004): Estimation is based on the centroids of a map object provided
	in boundary format (see function read.bnd() and shp2bnd()) as an additional argument named map within function sx(), e.g., map = MapBnd.
"gs",	Geosplines based on two-dimensional P-splines with a two-
"geospline"	dimensional first order random walk penalty for the parameters of the spline. Estimation is based on the coordinates of the centroids of the regions of a map object provided in boundary format (see func- tion read.bnd() and shp2bnd()) as an additional argument named map (see above).
"mrf", "spatial"	Markov random fields (Fahrmeir et al. 2004): Defines a Markov random field prior for a spatial covariate, where geographical information is provided by a map object in boundary or graph file format (see function read.bnd(), read.gra() and shp2bnd()), as an additional argument named map (see above).
"bl", "baseline"	Nonlinear baseline effect in hazard regression or multi-state models:
or , baseline	Defines a P-spline with second order random walk penalty for the parameters of the spline for the log-baseline effect $\log(\lambda(time))$.
"factor"	Special BayesX specifier for factors, especially meaningful if method = "STEP", since the factor term is then treated as a full term, which is either included or removed from the model.
"ridge", "lasso",	Shrinkage of fixed effects: Defines a shrinkage-prior for the corre-
"nigmix"	sponding parameters γ_j , $j=1,\ldots,q,\ q\geq 1$ of the linear effects
migmin.	x_1, \ldots, x_q . There are three priors possible: ridge-, lasso- and normal mixture of inverse gamma prior.
"re", "random",	Gaussian i.i.d. Random effects of a unit or cluster identification co-

Table 4: Possible ${\bf BayesX}$ model terms within function ${\tt sx()}$.

named map needs to be provided to sx(). For example, the map of Zambia in the geokriging term in Section 2 is included with sx(district, bs = "gk", map = ZambiaBnd). The map argument can be an object of class "SpatialPolygonsDataFrame" (Pebesma and Bivand 2005; Bivand, Pebesma, and Gómez-Rubio 2013) or an object of class "bnd". The latter is essentially a named list of the map's polygons which is the format required by BayesX for its computations. In case a "SpatialPolygonsDataFrame" is supplied it is transformed internally to such a polygon list which is employed for all further computations. Furthermore, "bnd" objects can be created directly using functions from the R package BayesX of Kneib et al. (2014): read.bnd() and shp2bnd() create "bnd" objects from text files or shapefiles (using package shapefiles, Stabler 2013), respectively. For MRF terms, it is possible to supply the whole map as outlined above but it suffices to supply the corresponding neighborhood information. Internally, BayesX uses a list specification of neighbors which is captured in objects of class "gra" that can be created by read.gra() and bnd2gra(). Improvements in the handling of spatial information — especially by leveraging more functionality from the sp family of packages — are planned for future versions of R2BayesX.

Some care is warranted for the identifiability of varying coefficients terms. The standard in **BayesX** is to center nonlinear main effects terms around zero whereas varying coefficient terms are not centered. This makes sense since main-effects nonlinear terms are not identifiable (with an intercept in the model) and varying coefficients terms are usually identifiable. However, there are situations where a varying coefficients term is not identifiable. Then the term must be centered. Since centering is not automatically accomplished it has to be enforced by the user by adding option center = TRUE in function sx(). To give an example, the varying coefficient terms in $\eta = \ldots + g_1(z_1)z + g_2(z_2)z + \gamma_0 + \gamma_1 z + \ldots$ are not identified, whereas in $\eta = \ldots + g_1(z_1)z + \gamma_0 + \ldots$, the varying coefficient term is identifiable. In the first case, centering is necessary, in the second case, it is not.

6. STAR models in practice

The focus of this section is on demonstrating the various features of the **R2BayesX** package. Therefore, the examples provided reconsider analyses from Brezger *et al.* (2005) and Fahrmeir *et al.* (2013). The presented datasets have been added to package **R2BayesX**, ensuring straightforward reproducibility of the following code. In the first example, a Gaussian regression model is estimated using Markov chain Monte Carlo simulation. The second example covers estimation based on mixed-model technology, where a cumulative threshold model is employed for an ordered response variable (see Fahrmeir and Tutz 2001, and Kneib and Fahrmeir 2006 for cumulative threshold models). The last example illustrates the approach of the stepwise algorithm for model and variable selection.

6.1. Childhood malnutrition in Zambia: Analysis with MCMC

This analysis has already been conducted by Kandala *et al.* (2001) and has also been used as a demonstrating example in Brezger *et al.* (2005). Stunting is one of the leading drivers of a number of problems developing countries are faced with, for instance, a direct consequence of stunting is a high mortality rate. Here, the primary interest is to model the dependence of stunting of newborn children, with an age ranging from 0 to 5 years, on covariates such as the body mass index of the mother, the age of the child and others presented in Table 5.

Variable	Description
stunting	Standardized Z-score for stunting.
mbmi	Body mass index of the mother.
agechild	Age of the child in months.
district	District where the mother lives.
memployment	Mother's employment status with categories 'yes' and 'no'.
meducation	Mother's educational status with categories for no education
	or incomplete primary 'no', complete primary but incom-
	plete secondary 'primary' and complete secondary or higher
	'secondary'.
urban	Locality of the domicile with categories 'yes' and 'no'.
gender	Gender of the child with categories 'male' and 'female'.

Table 5: Variables in the dataset on childhood malnutrition in Zambia.

The response stunting is standardized in terms of a reference population, i.e., in this dataset stunting for child i is represented by

$$\mathtt{stunting}_i = \frac{AI_i - m}{\sigma},$$

where AI refers to a child's anthropometric indicator (height at a certain age in our example), while m and σ correspond to the median and the standard deviation in the reference population, respectively.

Following Kandala et al. (2001), we estimate a structured additive regression model with predictor

$$\eta = \gamma_0 + \gamma_1$$
memploymentyes + γ_2 urbanno + γ_3 genderfemale + γ_4 meducationno + γ_5 meducationprimary + f_1 (mbmi) + f_2 (agechild) + f_{str} (district) + f_{unstr} (district) (2)

where memploymentyes is the deviation (effect) coded version of covariate memployment, generated with function contr.sum() by setting the contrasts argument of the factor variable, i.e., memploymentyes contains of values -1, corresponding to 'yes', and 1, 'no' respectively, likewise for covariates genderfemale, urbanno, meducationno and meducationprimary. As mentioned in the introduction, functions f_1 and f_2 of the continuous covariates agechild and mbmi are assumed to have a possibly nonlinear effect on stunting and are therefore modeled with P-splines. Furthermore, the spatial effect is decomposed into a structured effect f_{str} , modeled by a Gaussian Markov random field, and an unstructured effect f_{unstr} , using a random effects term for the districts in Zambia.

The data for this analysis is provided in the R2BayesX package and can be loaded with

R> data("ZambiaNutrition", package = "R2BayesX")

Since function f_{str} is modeled by a Markov random fields term, **BayesX** needs information about the district neighborhood structure, which e.g., is enclosed in the file

```
R> data("ZambiaBnd", package = "R2BayesX")
```

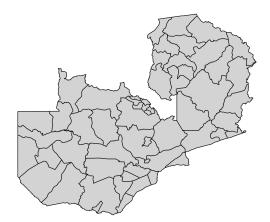


Figure 2: Example on childhood malnutrition: A simple map of the districts in Zambia.

The object ZambiaBnd has class "bnd" and is basically a list() of polygon matrices, with x-and y-coordinates of the boundary points in the first and second column respectively. With the information of the boundary file BayesX may compute an appropriate adjacency matrix, allowing for a smoothly varying effect of the neighboring regions. In addition, "bnd" objects can be used to calculate centroids of polygons to estimate smooth bivariate effects of the resulting coordinates (e.g., using the "geokriging" option in Section 2, also see Section 6.2 for another example). There is a generic plotting method implemented for objects of class "bnd", which essentially calls function plotmap(). E.g., a simple map, as shown in Figure 2, of the districts in Zambia is drawn by typing

R> plot(ZambiaBnd)

Having loaded the necessary files, the model formula is specified with

As mentioned above, the structured spatial effect is now modeled as a Markov random field (option "mrf"), while in Section 2 we used the region centroids to model a smooth spatial effect applying (geo)kriging. The model is then fitted using MCMC by calling

Argument iterations, burnin and step set the number of iterations of the MCMC simulation, the burnin period, which will be removed from the generated samples, and the step length for which samples should be stored, i.e., if step = 10, every 10th sampled parameter will be saved. In most applications 12000 iterations should be enough for a valid fit with sufficiently small autocorrelations of stored parameters, at least in the model building stage. However, it is crucial to inspect the sampled parameters and autocorrelation functions to check the mixing behavior (see below). Moreover, it is generally advisable to specify a higher

number of iterations for the final model that appears in publications. Argument seed sets the state of the random number generator in **BayesX** for exact reproducibility of the model fit.

After the model has been successfully fitted, summary statistics of the MCMC estimated model object may be printed with

R> summary(zm)

Call:

```
bayesx(formula = f, data = ZambiaNutrition, family = "gaussian",
   method = "MCMC", iterations = 12000, burnin = 2000, step = 10,
   seed = 123)
```

Fixed effects estimation results:

Parametric coefficients:

	Mean	Sd	2.5%	50%	97.5%
(Intercept)	0.1052	0.0497	0.0060	0.1059	0.1989
memploymentno	-0.0076	0.0139	-0.0356	-0.0071	0.0206
urbanno	-0.0897	0.0221	-0.1340	-0.0894	-0.0474
genderfemale	0.0583	0.0129	0.0338	0.0584	0.0833
meducationno	-0.1736	0.0277	-0.2256	-0.1731	-0.1203
meducationprimary	-0.0614	0.0255	-0.1125	-0.0623	-0.0128

Smooth terms variances:

```
Mean Sd 2.5% 50% 97.5% Min Max sx(agechild) 0.0059 0.0057 0.0012 0.0043 0.0193 0.0005 0.0701 sx(district):mrf 0.0355 0.0185 0.0100 0.0319 0.0823 0.0035 0.1317 sx(mbmi) 0.0019 0.0025 0.0003 0.0011 0.0079 0.0002 0.0319
```

Random effects variances:

```
Mean Sd 2.5% 50% 97.5% Min Max sx(district):re 0.0073 0.0059 0.0006 0.0056 0.0215 0.0002 0.0374
```

Scale estimate:

```
Mean Sd 2.5% 50% 97.5% Sigma2 0.8019 0.0165 0.7718 0.8017 0.8342
```

```
N = 4847 burnin = 2000 DIC = 4903.827 pd = 50.69312 method = MCMC family = gaussian iterations = 12000 step = 10
```

The summary typically includes mean, standard deviation and quantiles of sampled linear effects, smooth terms variances and random effects variances, as well as goodness of fit criteria and some other information about the model. The estimated effects for covariates agechild and mbmi may then be visualized with

```
R> plot(zm, term = c("sx(mbmi)", "sx(agechild)"))
```

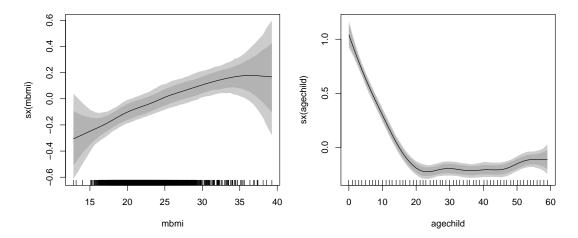


Figure 3: Example on childhood malnutrition: Effect of the body mass index of the child's mother and of the age of the child together with pointwise 80% and 95% credible intervals.

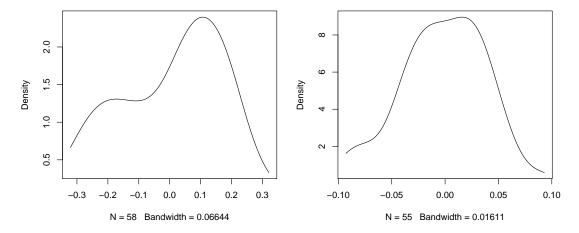


Figure 4: Example on childhood malnutrition: Kernel density estimates of the posterior means of coefficients for all regions of the structured, left panel, and the unstructured spatial effect, right panel respectively.

and are shown in Figure 3. The interpretations of both terms are essentially unchanged compared to the simpler model considered in Section 2: The age of the child has a larger effect on stunting while mother's BMI could also be modeled approprietly by a linear term.

A visual representation of the posterior means for the structured and unstructured spatial effects, respectively, can be obtained in two ways: via kernel density estimates or using shaded maps. The former can be obtained by the plain plot function yielding both panels of Figure 4:

R> plot(zm, term = c("sx(district):mrf", "sx(district):re"))

Note that here the term labels have been extended by their respective basis specifications (mrf and re) to make the labels unique. Equivalently, term can also be specified by the corresponding index (based on the ordering in the model formula), e.g., term = c(7, 8) (see Appendix B for more details). In Figure 4, the kernel densities reveal the general form of the random effects distributions which are assumed to follow a Gaussian distribution. The

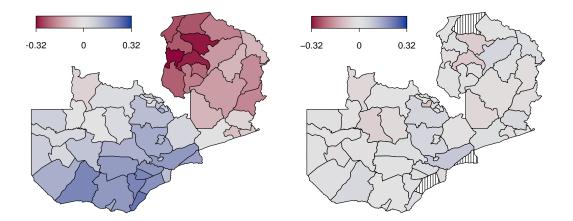


Figure 5: Example on childhood malnutrition: Estimated mean effect of the structured spatial effect (left panel), together with the unstructured spatial effect using the color and legend scaling of the structured effect (right panel).

range of the estimated random spatial effect is much smaller than the range of the structured spatial effect, indicating that model fit improvement by including random effects that account for unobserved spatial heterogeneity of the regions in Zambia, is relatively low. This is also supported by the comparatively low variance estimate of the random effects term given in the model summary above.

Alternatively, to view the spatial structure of the correlated effect the plot function can be used in combination with the boundary object ZambiaBnd yielding the map effect plot in the left panel of Figure 5:

```
R> plot(zm, term = "sx(district):mrf", map = ZambiaBnd)
```

As a default the districts of Zambia are colored in a symmetrical range within the estimated $\pm \max(|\text{posterior mean}|)$ of the corresponding effect. In many situations the visual impression of the colored map is problematic. This is primarily the case if there are some districts with extraordinarily high posterior means compared to the rest of the districts. Then the map is dominated by the colors of these outlying districts. A more informative map may be obtained by restricting the range of the plotting area using the range option. For the Zambia data the corresponding random effects are comparably symmetric and without outlying districts such that the plot function with default options produces fairly informative maps. To demonstrate the range option we draw the unstructured random effect and the legend range within the same range as the structured random effect, yielding the right panel of Figure 5:

```
R> plot(zm, term = "sx(district):re", map = ZambiaBnd,
+ range = c(-0.32, 0.32), lrange = c(-0.32, 0.32))
```

Using the same scale for both the structured and the unstructured effect is useful for comparison. In most cases one of the two effects clearly dominates the other. In our case the structured spatially correlated effect clearly exceeds the unstructured effect.

In addition, care has to be taken interpreting structured and unstructured spatial effects. As has been shown in Fahrmeir and Lang (2001) the unstructured and the structured spatial

effects is estimated satisfactorily. This means in practice that only the complete spatial effect should be interpreted and nothing (or not much) can be said about the relative importance of both effects. Exception are cases where one of both effects (either the unstructured or the structured effect) is estimated practically zero and the other effect clearly dominates as shown in this example. Whenever a structured and unstructured spatial effect is estimated it is possible to plot the sum of both effects by extending the corresponding term name with ":total", i.e., similar to the ":mrf" and ":re" extension in the above.

In addition, autocorrelation functions may be drawn, e.g., for the variance samples of term sx(mbmi), by typing

```
R> plot(zm, term = "sx(mbmi)", which = "var-samples", acf = TRUE)
```

For MCMC post estimation diagnosis, it is also possible to extract sampling paths of parameters with function samples(), or to plot the samples directly. For instance, coefficient sampling paths for term sx(mbmi) are displayed with

```
R> plot(zm, term = "sx(mbmi)", which = "coef-samples")
```

see Figure 6. The plot of sampled parameters should ideally show white noise, i.e., more or less uncorrelated samples that show no particular pattern. In our case the samples are exactly as they should be. The maximum autocorrelation of all sampled parameters in the model are displayed with

```
R> plot(zm, which = "max-acf")
```

Autocorrelations for all lags should be close to zero as is mostly the case in our example. See Figure 7, for the autocorrelation plots. The plot of maximum autocorrelations over all model parameters suggests to use a larger number of iterations in a final run (e.g., 22000 or even 32000 iterations) to improve the mixing behavior, i.e., the number of iterations should be chosen such that the samples after thinning are (nearly) uncorrelated.

Convergence can also be checked, e.g., by running multiple chains or cores. A model using two chains can be estimated by

The returned object zm2 now contains of two separate models, and is also of class "bayesx", for which all summary, plotting an extractor functions can be used. Now, to further analyze convergence, the user can extract the samples for certain parameters and terms with the extractor function samples()

```
R> zs <- samples(zm2, term = "linear-samples")</pre>
```

¹For convenience, the plot of the total spatial effect is not shown here.

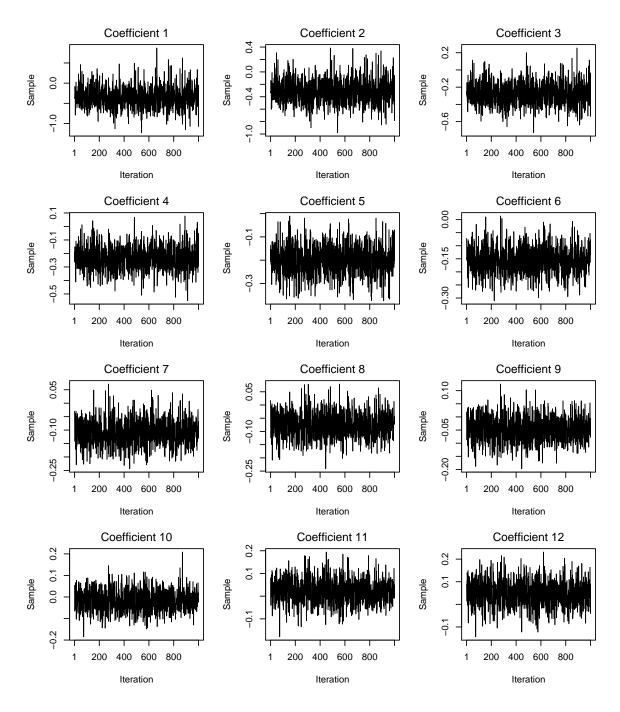


Figure 6: Example on childhood malnutrition: Sampling paths of the first 12 coefficients of term sx(mbmi).

In this case we only extract samples for the parametric modeled terms. Per default, function samples() returns an object of class "mcmc" using a single chain and core or of class "mcmc.list" when multiple chains or cores are used. One convergence diagnostic function, as implemented in the package **coda**, is the Gelman and Rubin's convergence diagnostic, that can then be computed e.g. by

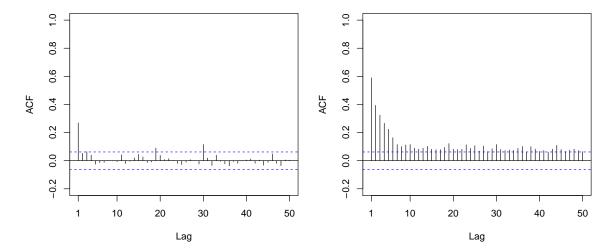


Figure 7: Example on childhood malnutrition: Autocorrelation function of the samples of the variance parameter of term sx(mbmi) (left panel) and maximum autocorrelation of all parameters of the model (right panel).

R> gelman.diag(zs, multivariate = TRUE)

Potential scale reduction factors:

	Point est.	Upper C.I.
Intercept	1.002	1.010
memploymentno	1.001	1.005
urbanno	1.003	1.004
genderfemale	1.004	1.016
meducationno	0.999	0.999
meducationprimary	1.008	1.037

Multivariate psrf

1.01

In some situations problems may occur during processing of the <code>BayesX</code> binary, that are not automatically detected by the main model fitting function <code>bayesx()</code>. Therefore the user may inspect the log-file generated by the binary in two ways: Setting the option <code>verbose = TRUE</code> in <code>bayesx.control()</code> (used within the model fitting function <code>bayesx()</code>, as mentioned in Section 5, note that control arguments in <code>bayesx()</code> can be passed directly to <code>bayesx.control()</code> by the dots "..." argument) will print all information produced by <code>BayesX</code> simultaneously at runtime. The option is especially helpful if <code>BayesX</code> fails in the estimation of the model.

Another way to obtain the log-file is to use function bayesx_logfile() if BayesX successfully finished processing. In this example the log-file may be printed with

R> bayesx_logfile(zm)

- > bayesreg b
- > map ZambiaBnd

```
> ZambiaBnd.infile using /tmp/Rtmpa3Z6WF/bayesx/ZambiaBnd.bnd
NOTE: 57 regions read from file /tmp/Rtmpa3Z6WF/bayesx/ZambiaBnd.bnd
> dataset d
> d.infile using /tmp/Rtmpa3Z6WF/bayesx/bayesx.estim.data.raw
NOTE: 14 variables with 4847 observations read from file
/tmp/Rtmpa3Z6WF/bayesx/bayesx.estim.data.raw
> b.outfile = /tmp/Rtmpa3Z6WF/bayesx/bayesx.estim
> b.regress stunting = mbmi(psplinerw2,nrknots=20,degree=3) +
    agechild(psplinerw2,nrknots=20,degree=3) + district(spatial,map=ZambiaBnd) +
    district(random) + memploymentyes + urbanno + genderfemale + meducationno +
    meducationprimary, family=gaussian iterations=12000 burnin=2000 step=10
    setseed=123 predict using d
NOTE: no observations for region 11
NOTE: no observations for region 84
NOTE: no observations for region 96
BAYESREG OBJECT b: regression procedure
GENERAL OPTIONS:
  Number of iterations:
                        12000
```

RESPONSE DISTRIBUTION:

Thinning parameter:

Burn-in period:

Family: Gaussian

Number of observations: 4847

Number of observations with positive weights: 4847

2000

10

Response function: identity Hyperparameter a: 0.001 Hyperparameter b: 0.001

To simplify matters only a fragment of the log-file is shown in the above. The log-file typically provides information on the used data, model specifications, algorithms and possible error messages.

6.2. Forest health dataset: Analysis with REML

The dataset on forest health comprises information on the defoliation of beech trees, which serves as an indicator of overall forest health here. The data were collected annually from 1980 to 1997 during a project of visual inspection of trees around Rothenbuch, Germany, see Göttlein and Pruscha (1996), and is discussed in detail in Fahrmeir et al. (2013). In this example, the percentage rate of defoliation of each tree is aggregated into three ordinal

Variable	Description
id	Tree location identification number.
year	Year of census.
defoliation	Percentage of tree defoliation in three ordinal categories: '<
	12.5% ', ' $12.5\% \le \text{defoliation} < 50\%$ ', ' $\ge 50\%$ '.
age	Age of stands in years.
canopy	Forest canopy density in percent.
inclination	Slope inclination in percent.
elevation	Elevation (meters above sea level).
soil	Soil layer depth in cm.
ph	Soil pH at 0–2cm depth.
moisture	Soil moisture level with categories 'moderately dry', 'mod-
	erately moist' and 'moist or temporarily wet'.
alkali	Proportion of base alkali-ions with categories 'very low',
	'low', 'high' and 'very high'.
humus	Humus layer thickness in cm.
stand	Stand type with categories 'deciduous' and 'mixed'.
fertilized	Fertilization applied with categories 'yes' and 'no'.

Table 6: Variables in the forest health dataset.

categories, which are modeled in terms of covariates characterizing the stand and site of a tree. In addition, temporal and spatial information is available, see also Table 6.

Similar to Fahrmeir et al. (2013), we start with a threshold model and cumulative logit link, with $P(\texttt{defoliation}_{it} \leq r)$ of tree i at time t, for response category r = 1, 2, and the additive predictor

$$\eta_{it}^{(r)} \ = \ f_1(\texttt{age}_{it}) + f_2(\texttt{inclination}_i) + f_3(\texttt{canopy}_{it}) + f_4(\texttt{year}) + f_5(\texttt{elevation}_i) + \mathbf{x}_{it}^\top \boldsymbol{\gamma}$$

where f_1, \ldots, f_5 are possibly nonlinear smooth functions of the continuous covariates and $\mathbf{x}_{it}^{\mathsf{T}} \boldsymbol{\gamma}$ comprises covariates with parametric effects using deviation (effect) coding for factor covariates.

To estimate the model within R the data is loaded and the model formula specified with

The covariates entering nonlinearly are again modeled by P-splines. The model is then fitted applying REML by assigning a cumulative logit model and calling

After the estimation process has converged, the estimated effects of the nonparametric modeled terms may be visualized by

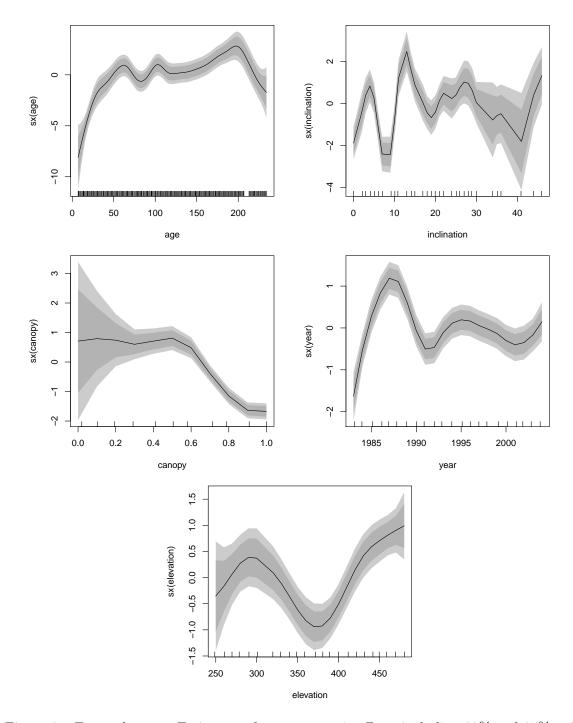


Figure 8: Forest damage: Estimates of nonparametric effects including 80% and 95% pointwise confidence intervals of the model without the spatial effect.

The results are shown in Figure 8 and appear to be rather unintuitive. In particular, the effect of the covariate age on defoliation seems to be non-monotonic with low defoliation levels for both younger and older trees. Similarly, the effect of elevation is very non-monotonic with high defoliation for both low and high elevations. Finally, the extremely wiggly estimate of inclination is hardly interpretable. Therefore, Göttlein and Pruscha (1996) extend the

model by a spatial effect, modeled by a two dimensional geospline term of the tree locations. The tree x- and y-coordinates are calculated by the centroid positions of tree polygons given by the boundary map file BeechBnd. We can update the model by adding a "gs" effect:

Note that argument nrknots is set to 20 (default is 8) to obtain a sufficiently flexible geospline that replicates the analysis of Fahrmeir *et al.* (2013). The associated model information criteria are:

```
R> BIC(fm1, fm2)

df BIC
fm1 59.9714 2016.04
fm2 94.8222 1930.06

R> GCV(fm1, fm2)

df GCV
fm1 59.9714 0.816340
fm2 94.8222 0.610199
```

This clearly indicates a better fit by modeling the spatial effect of tree locations. The summary statistics for both models gives:

```
R> summary(fm1)
```

Call:

```
bayesx(formula = f, data = ForestHealth, family = "cumlogit",
    method = "REML")
```

Fixed effects estimation results:

Parametric coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
theta_1	-4.3485	1.5039	-2.8914	0.0039	**
theta_2	0.7500	1.5156	0.4948	0.6208	
standmixed	-0.6175	0.1044	-5.9178	<2e-16	***
fertilizedno	0.5362	0.1901	2.8208	0.0048	**
humus[0cm, 1cm]	-0.1407	0.1648	-0.8536	0.3934	
humus(1cm, 2cm]	0.4421	0.1682	2.6289	0.0086	**
humus(2cm, 3cm]	0.0975	0.1793	0.5439	0.5866	
humus(3cm, 4cm]	0.0771	0.2307	0.3341	0.7383	
moisturemoderately dry	-0.7569	0.2088	-3.6246	0.0003	***
moisturemoderately moist	0.3067	0.1418	2.1625	0.0307	*

```
      alkalivery low
      1.1612
      0.2482
      4.6793
      <2e-16 ***</td>

      alkalilow
      -0.3889
      0.1881
      -2.0680
      0.0388 *

      alkalihigh
      -0.9853
      0.2242
      -4.3957
      <2e-16 ***</td>

      ph
      -0.8074
      0.3021
      -2.6728
      0.0076 **

      soil
      -0.0470
      0.0104
      -4.5008
      <2e-16 ***</td>
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

Smooth terms:

	Variance	Smooth Par.	df	Stopped
sx(age)	4.9911	0.2004	12.3322	0
sx(canopy)	0.0527	18.9743	4.7092	0
<pre>sx(elevation)</pre>	0.0668	14.9682	5.0563	0
<pre>sx(inclination)</pre>	25.8453	0.0387	14.4449	0
sx(year)	0.2971	3.3664	8.4287	0

N = 1793 df = 59.9714 AIC = 1686.69 BIC = 2016.04 logLik = -783.375 GCV = 0.81634 method = REML family = cumlogit

R> summary(fm2)

Call:

```
bayesx(formula = defoliation ~ stand + fertilized + humus + moisture +
    alkali + ph + soil + sx(age) + sx(inclination) + sx(canopy) +
    sx(year) + sx(elevation) + sx(id, bs = "gs", map = BeechBnd,
    nrknots = 20), data = ForestHealth, family = "cumlogit",
    method = "REML")
```

Fixed effects estimation results:

Parametric coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
theta_1	-1.8244	2.0034	-0.9106	0.3626	
theta_2	4.5302	2.0421	2.2184	0.0267	*
standmixed	-0.1778	0.2269	-0.7835	0.4335	
fertilizedno	0.5816	0.4977	1.1685	0.2428	
humus[Ocm, 1cm]	-0.3371	0.2004	-1.6817	0.0928	
humus(1cm, 2cm]	0.2453	0.1951	1.2576	0.2087	
humus(2cm, 3cm]	0.1656	0.2066	0.8014	0.4230	
humus(3cm, 4cm]	0.2205	0.2578	0.8552	0.3926	
moisturemoderately dry	-0.7054	0.5450	-1.2943	0.1957	
moisturemoderately moist	-0.0765	0.3899	-0.1961	0.8446	
alkalivery low	0.9401	0.6297	1.4929	0.1357	
alkalilow	-0.3564	0.4866	-0.7324	0.4640	
alkalihigh	-0.3869	0.5608	-0.6899	0.4904	
ph	-0.3033	0.3611	-0.8399	0.4011	
soil	-0.0072	0.0281	-0.2553	0.7985	

```
Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```

Smooth terms:

	Variance	Smooth Par.	df	Stopped
sx(age)	3.8455	0.2600	10.9703	0
sx(canopy)	0.0179	55.8909	3.2481	0
<pre>sx(elevation)</pre>	0.0002	5203.4900	1.0280	1
sx(id)	56.3986	0.0177	53.6092	0
<pre>sx(inclination)</pre>	0.0103	97.4621	1.8657	0
sx(vear)	0.5220	1.9158	9.1008	0

```
N = 1793 df = 94.8222 AIC = 1409.33 BIC = 1930.06 logLik = -609.84 GCV = 0.610199 method = REML family = cumlogit
```

Most of the parametric modeled terms in the second model now have an insignificant effect on tree defoliation, with similar findings for covariates inclination and elevation (where the pointwise 95% credible intervals cover the zero line). However, the estimate of the age effect seems to be improved in terms of monotonicity, see Figure 9.

A kernel density plot of the estimated spatial effect is then obtained by

```
R> plot(fm2, term = "sx(id)", map = FALSE)
```

The effect may also be visualized either using a 3d perspective plot, an image/contour plot or a map effect plot using the boundary file BeechBnd with

```
R> plot(fm2, term = "sx(id)", map = BeechBnd)
```

Both the kernel density and map effect plot are shown in the first two panels of Figure 10. In this example the coloring of the plot is strongly influenced by a few very high and low values. In addition, the size of the polygon areas is relatively small and makes it difficult to examine the effect. Therefore, it is helpful to plot a smooth interpolated map of the effect. The resulting map in the bottom panel of Figure 10 is created by:

```
R> plot(fm2, term = "sx(id)", map = BeechBnd,
+ interp = TRUE, outside = TRUE)
```

where argument interp specifies if interpolation of estimated effects should be applied and argument outside if values outside the polygon areas should be shown (see also Appendix B). Plotting interpolated values now leads to a better representation of the effect.

In summary, the results identify a strong influence of the spatial effect on the overall model fit, indicating that a clear splitting of location-specific covariates and the spatial effect is hardly possible in this example.

6.3. Childhood malnutrition in Zambia: Analysis with STEP

To illustrate the implemented methodology for simultaneous selection of variables and smoothing parameters, we proceed with the dataset on malnutrition in Zambia of Section 6.1. In

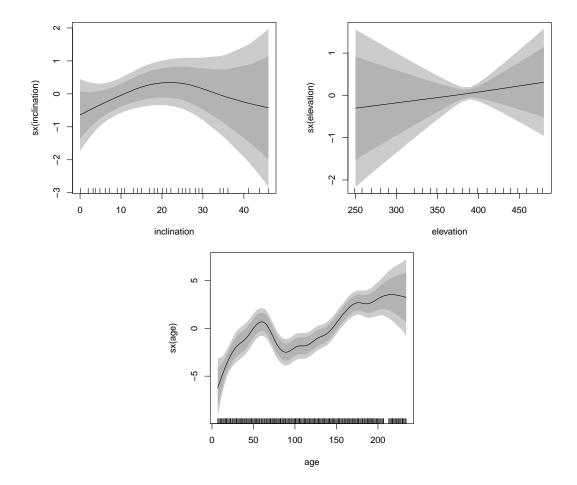


Figure 9: Forest damage: Estimated effects of covariates inclination, elevation and age, including 80% and 95% point-wise confidence intervals, of the model including the spatial effect.

this example, the structured additive predictor (2) contains two continuous covariates mbmi and agechild, that are assumed to have a possibly nonlinear effect on the response stunting and are modeled with P-splines. However, to assess whether this is really necessary the corresponding linear effect is also considered using the selection algorithm in **BayesX**. Additionally, for each variable and function, the implemented procedures decide if a term is included or removed from the model. To estimate the model applying the option method = "STEP", we use the same model formula of Section 6.1 and call

where argument algorithm chooses the selection algorithm and startmodel the start model for variable selection, see also Table 2 for all possible options. Usually the selected final model

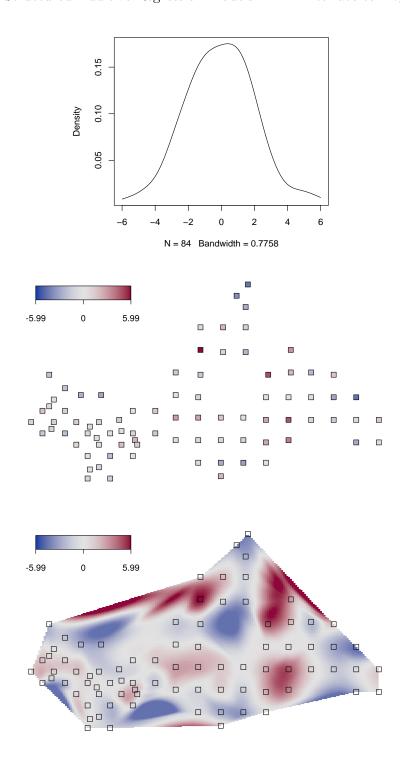


Figure 10: Forest damage: Kernel density estimate of the spatial effect (top panel), together with a map effect plot (middle panel), and a map effect plot applying smooth spatial interpolation (bottom panel).

is pretty much insensitive with respect to the selection algorithm and startmodel. However, it is generally of interest to assess the dependence of results on the selection algorithm and

the startmodel. The summary statistics of the final selected model are then provided with

```
R> summary(zms)
```

Call:

```
bayesx(formula = f, data = ZambiaNutrition, family = "gaussian",
   method = "STEP", algorithm = "cdescent1", startmodel = "empty",
   seed = 123)
```

Fixed effects estimation results:

Parametric coefficients:

```
Mean Sd 2.5% 50% 97.5%
(Intercept)
            -0.4863 NA
                          NA
                              NA
urbanno
             -0.0945 NA
                          NA
                              NA
                                    NA
genderfemale 0.0589 NA
                              NA
                                    NA
                          NA
meducation_0 -0.1087 NA
                              NA
                                    NA
                          NA
meducation_2 0.2977 NA
                          NA NA
                                    NA
mbmi
              0.0209 NA
                          NA NA
                                    NA
```

Smooth terms:

```
lambda df

sx(agechild) 15.1687 10.981

sx(district) 7.5775 24.391

sx(district):re 35.8077 17.835
```

Scale estimate: 0.7897

```
N = 4847 AIC_imp = -1024.37 method = STEP family = gaussian
```

Thus, the results are similar to those from model zm in Section 6.1. However, the variable memployment is removed from the model and variable mbmi is modeled by a linear effect.

By default, the columns sd, 2.5%, 50% and 97.5% from a "STEP" fit contain no values, likewise for the estimated random and smooth effects. The posterior quantiles may be computed if argument CI in function bayesx.control() is specified. E.g., conditional confidence bands can be calculated conditional on the selected model, i.e., they are computed for selected variables and functions only. The computation of conditional confidence bands is based on an MCMC-algorithm subsequent to the selection procedure. For the selection of a model with a subsequent computation of conditional confidence bands the user may type

```
R> zmsccb <- bayesx(f, family = "gaussian", method = "STEP",
+ algorithm = "cdescent1", startmodel = "empty", CI = "MCMCselect",
+ iterations = 10000, step = 10, seed = 123, data = ZambiaNutrition)</pre>
```

which results in the following summary

```
R> summary(zmsccb)
```

Call:

```
bayesx(formula = f, data = ZambiaNutrition, family = "gaussian",
   method = "STEP", algorithm = "cdescent1", startmodel = "empty",
   CI = "MCMCselect", iterations = 10000, step = 10, seed = 123)
```

Fixed effects estimation results:

Parametric coefficients:

```
        Mean
        Sd
        2.5%
        50%
        97.5%

        (Intercept)
        -0.4815
        0.0982
        -0.6737
        -0.4813
        -0.2922

        urbanno
        -0.0939
        0.0239
        -0.1373
        -0.0949
        -0.0464

        genderfemale
        0.0587
        0.0124
        0.0342
        0.0590
        0.0824

        meducation_0
        -0.1087
        0.0297
        -0.1690
        -0.1098
        -0.0516

        meducation_2
        0.2961
        0.0661
        0.1724
        0.2934
        0.4223

        mbmi
        0.0207
        0.0043
        0.0123
        0.0207
        0.0287
```

Smooth terms:

```
lambda df

sx(agechild) 15.1687 10.981

sx(district) 7.5775 24.391

sx(district):re 35.8077 17.835
```

Scale estimate: 0.7897

```
N = 4847 DIC = 4966.286 pd = 59.60813 AIC_imp = -1024.37 method = STEP family = gaussian iterations = 10000 step = 10
```

It is also possible to obtain unconditional confidence bands by setting CI = "MCMCbootstrap", which additionally considers the uncertainty due to model selection. The model is specified e.g. by

```
R> zmsccb2 <- bayesx(f, family = "gaussian", method = "STEP",
+ CI = "MCMCbootstrap", bootstrapsamples = 99, iterations = 10000,
+ step = 10, seed = 123, data = ZambiaNutrition)</pre>
```

The bootstrap approach for obtaining unconditional confidence bands also provides the relative frequencies each model term has been selected during the bootstrap iterations. The frequency tables can be extracted using function term.freqs(), e.g. for the effect of the body mass index of the mother the frequency table is extracted by

R> term.freqs(zmsccb2, term = "sx(mbmi)")

	df	lambda	frequency	selected
1	1.00000	-1.00000	73	+
2	1.95385	6718.75000	13	_
3	3.02125	1313.07000	4	-
4	3.95243	456, 20900	2	_

```
5
    5.95180
               80.80170
                                   1
6
    7.03855
               37.43130
                                  3
7
    7.97073
               20.50530
                                   1
8
    8.99663
               11.04710
                                   1
9
    9.98339
                6.26908
                                   1
10 11.02430
                3.50045
                                   1
```

showing that a linear representation of that term with one degrees of freedom has been selected 73 times by the algorithm, i.e., as in the above, the linear effect for variable mbmi is selected in the final model.

Another variation of this model would be to start from a "userdefined" instead of an "empty" startmodel (see also Table 2 for further options). In the "userdefined" case, it may be reasonable to start with a the initial degrees of freedom (complexity or roughness) in the search for the nonlinearly modeled terms can be supplied. For example, the starting values for the degrees of freedom of the P-spline, spatial and random effect terms can be specified via

7. Summary

The R package R2BayesX provides an interface to the standalone software package BayesX for estimation of structured additive regression (STAR) models via MCMC, REML, or stepwise selection. The interface has the usual "look & feel" of regression modeling functions in R with a formula-based fitting function bayesx() along with suitable methods such as summary() and plot(). Adapting functionality from the mgcv package, the package allows for specification of regressions with smooth terms via the sx() constuctor function. This is implemented using mgcv's smooth term constructor s() but facilitates specification of BayesX-specific terms along with corresponding optional control arguments. Moreover, the software design is modular enabling the import of already existing BayesX fitted-model files or the execution of previously generated BayesX program files from R. For post-estimation analysis and graphical inspection, the suite of methods allows for extraction of summary statistics and fitted model term objects as well as generation of 2d, 3d, image, and map effect plots, amongst others.

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A. BayesXsrc: Packaging the BayesX C++ sources for R

BayesX was originally developed under the Borland C++ compiler and is distributed as a Windows application with a Java-based user interface. Since version 2.0, it also offers a command line version comprising the interpreter and modules for computations. The sources have been modified to be compliant with the GNU Compiler Collection (GCC), and the software was ported to run on Linux, Mac OS X, Windows and several BSDs.

Our objective with the R package **BayesXsrc** is to offer R users a convenient way to download, build, and install the open-source **BayesX** software as if this were an ordinary R package, and for offering prebuilt binary versions of **BayesX** through the CRAN build servers for major R platforms.

To accomplish this goal, <code>BayesXsrc</code> comes with a tiny R package hull, within which the <code>BayesX</code> sources for the command line version are embedded. In order to compile the <code>BayesX</code> sources with the R build system (e.g., via R CMD INSTALL), <code>Makefiles</code> under <code>src/</code> are utilized to compile the sources stored at <code>src/bayesxsrc</code>. The current source tree of <code>BayesX</code> requires a slightly different setting of compile flags for Windows which is achieved by using the two standard locations for R <code>Makefiles: src/Makefile.win</code> for Windows and <code>src/Makefile</code> otherwise. Since R 2.13.1 the package installation was enhanced to support non-standard installation of compiled code via an R installation script. If an R script <code>src/install.libs.R</code> is found, it will be executed after successful compilation. We make use of this feature to copy the binary executable to the package installation directory in an architecture-specific subfolder to support multi-architecture installations using R as a cross-platform portable install shell:

```
binary <- if(WINDOWS) "BayesX.exe" else "BayesX"
if(file.exists(binary)) {
  libarch <- if (nzchar(R_ARCH)) paste("libs", R_ARCH, sep = "") else "libs"
  dest <- file.path(R_PACKAGE_DIR, libarch)
  dir.create(dest, recursive = TRUE, showWarnings = FALSE)
  file.copy(binary, dest, overwrite = TRUE)
}</pre>
```

Since the executable exists in a designated location within the installed **BayesXsrc** package, we can run the command line version from within R via a single front-end function run.bayesx().

Note that the package hull of **BayesXsrc** (including inst/bayesxsrc) is maintained in a subversion (SVN) repository on R-Forge (at https://R-Forge.R-project.org/projects/bayesr/, along with **R2BayesX**). The source code will be updated if a new stable release of **BayesX** is available.

Although we do not effectively distribute an R package in the usual sense, we use the R package system as a cross-platform build system. The installation via an R package is an attractive alternative, in particular for software that aims to be embedded to R. Potential support for distribution and delivery of self-contained software in form of sources and prebuilt binaries via CRAN is very attractive to end users but also to smaller development teams (like us) that would otherwise have no resources for multi-platform builds and tests.

B. Options for the plot() method

Argument	Description
term	The term that should be plotted, either an integer or a character, e.g.,
	term = "sx(x)".
which	Choose the type of plot that should be drawn, possible options
	are: "effect", "coef-samples", "var-samples", "intcpt-samples",
	"hist-resid", "qq-resid", "scatter-resid", "scale-resid",
	"max-acf". Argument which may also be specified as integer, e.g.,
	which = 1. The first three arguments are all model term-specific. For
	the residual model diagnostic plot options which may be set with which
	= 5:8.
acf	If set to TRUE and which specifies samples to be plotted, the autocor-
	relation function of the samples are shown.
residuals	If set to TRUE, partial residuals may also be plotted if available.
rug	If set to TRUE, a rug() is added to the plot.
jitter	If set to TRUE, a jitter()ed rug() is added to the plot.
col.surface	The color of the surface, may also be a function, e.g., col.surface =
	heat.colors.
grid	The grid size of the surface(s).
image	If set to TRUE, an image.plot() is drawn.
contour	If set to TRUE, a contour() plot is drawn.
map	The map to be plotted, the map object must be a list of matrices
	with first column indicating the x-coordinate and second column the
	y-coordinate each, see also function polygon().
legend	If set to TRUE, a legend will be shown.
range	Specify the range of values the plot should be generated for, e.g., only
	values between -2 and 2 are of interest then range = $c(-2, 2)$.
interp	If set to TRUE, values will be smoothly interpolated.
outside	If set to TRUE, interpolated values outside the polygon areas will be
	plotted.
color	The colors for the legend, may also be a function, e.g., colors =
	heat.colors.
pos	The position of the legend, either a numeric vector, e.g., pos = c(0.1,
	0.2) will add the legend at the 10% point in the x-direction and at the
	20% point in the y-direction of the plotting window, may also be neg-
	ative, or one of the following: "bottomleft", "topleft", "topright"
	or "bottomright". Using function plotmap() option "right" is also valid.
lrange	Specifies the range of the legend.
symmetric	If set to TRUE, a symmetric legend will be drawn corresponding to the
·	$\pm \max(x)$ of values x that are used for plotting.

Table 7: Most important arguments of the plot() method for "bayesx" objects. The first block describes arguments of the plot() method itself, subsequent blocks arguments that are passed to plot2d(), plot3d(), plotmap(), and colorlegend(), respectively.

Objects of class "bayesx" returned either from function bayesx() or read.bayesx.output() have a method for the plot() generic. Depending on the structure of the "bayesx" object, the method identifies the various types of inherent model terms and applies one of the following implemented plotting functions: plot2d(), plot3d() or plotmap(). Using the method without further specifications will produce a plot of all estimated effects. For individual effect plots argument term is used. For MCMC estimated models argument which is useful to inspect sampling paths of coefficients, but also to view basic residual diagnostic plots. To build map effect plots using plotmap(), a map needs to be supplied to argument map. The map must be an object of "bnd" or "SpatialPolygonsDataFrame". Per default, similar to 2d plots, map effect plots are colored using a diverging color legend where the range is set symmetrical, e.g. according to the \pm max(|posterior mean|) of the effect. In this setting it is easier to distinguish between regions of large and no influence. The most important options of the plotting method are shown in Table 7, for a detailed description of all available arguments and options please see documentation of function plot.bayesx(), plot2d(), plot3d(), plotmap() and colorlegend().

C. Smooth term constructor functions

The main model term constructor function in **R2BayesX** is sx() (see also Section 4.2) which is simply a front-end to **mgcv**'s smooth term constructor function s() with **BayesX**-specific argument names and corresponding defaults. Due to this setup, s() (instead of sx()) can also be used directly in **bayesx**() model calls, facilitating specification of models in a way familiar to **mgcv** users. However, note that some arguments are named/defined somewhat differently in **mgcv** and **BayesX**; and due to the usage of different estimation methods not all settings that work well in one package necessarily work well in the other, too.

The smooth term construction typically proceeds in the following three steps:

- 1. sx() is called by the user in a formula. It takes BayesX-specific argument/option names and corresponding defaults, and maps them to argument names/values consistent with s() from mgcv.
- 2. s() is called by sx(), creating an object of class "xx.smooth.spec" where "xx" is the name of the basis type bs specified.
- 3. bayesx.construct() is called when the formula is parsed. If a method for objects of class "xx.smooth.spec" exists, this maps the mgcv-style arguments back to BayesX-style arguments, and subsequently creates a character string with the corresponding BayesX-interpretable command.

Although this entails mapping of arguments from **BayesX** style to **mgcv** style in Step 1 and back again in Step 3, this is worth the effort for two reasons: (a) **R2BayesX** does not have to reinvent a mechanism for storing information about smooth terms but simply leverages the **mgcv** system. (b) Expert users can skip Step 1 above and directly supply s() calls in bayesx() model formulas.

Function sx() has two arguments that directly correspond to s() arguments: bs for specifying the type of basis/term used, by for nesting of smooth terms. All remaining term specifications in sx() are passed through ... and depend on the basis type bs (and can be queried via

Basis/type bs	sx() argument/default	s()/te() equivalent
"ps"	nrknots = 20	k = nrknots + degree - 1 *
	degree = 3	m = c(degree - 1, order)
	order = 2	
	• • •	xt = list()
"te", te()	nrknots = 8	k = nrknots + degree - 1 *
	degree = 3	m = c(degree - 1, order)
	order = 2	
		xt = list()
"mrf"	map	<pre>xt = list(map,)</pre>
	• • •	

Table 8: Argument mapping and default specifications of smooth terms available in both sx() and s()/te(). Rows marked with * indicate that the s() default is different from the corresponding sx() default. The map argument for bs = "mrf" has to be a list of polygons and has no default in both sx() and s().

bayesx.term.options()). These ... arguments are mapped to the s() arguments k for the dimension of the basis, m for the (basis and) penalty order, and xt for extra information.

An overview of the argument mapping performed is shown in Table 8 for those smooth term types that can be estimated by bayesx() and gam() from mgcv: bs = "ps" for P-splines, bs = "mrf" for Markov random fields (MRFs), or a tensor product of two P-splines which can be created either as sx(x1, x2, bs = "te") or te(x1, x2, bs = "ps"). Some additional details and illustrations (using the Zambia malnutrition data) are provided below.

P-splines. The default smoothing splines employed in **R2BayesX** are P-splines. Hence,

R> bayesx.construct(sx(mbmi))

[1] "mbmi(psplinerw2,nrknots=20,degree=3)"

produces a P-spline term with degree 3 basis functions constructed from 20 inner knots. As this corresponds to 22 B-spline basis functions (of degree 3), the equivalent s() call would be s(mbmi, bs = "ps", k = 22). Note that the s() default k = 10 corresponds to a much lower-dimensional basis

R> bayesx.construct(s(mbmi, bs = "ps"))

[1] "mbmi(psplinerw2,nrknots=8,degree=3)"

Finally, note that the default for bs in s() is *not* "ps" but "tp" for thin-plate splines so that s(mbmi) cannot be used in bayesx()/bayesx.construct() and hence results in an error.

Tensor products. For two-dimensional smoothing, **BayesX** offers two-dimensional P-splines as bs = "te" where both marginal P-splines have the same degree and number of inner knots. In package **mgcv**, such terms can be constructed with the function te() which is hence also supported in **R2BayesX**. The following specifications lead to identical results when passed to bayesx.construct():

```
R> sx(mbmi, agechild, bs = "te")
R> te(mbmi, agechild, bs = "ps", k = 7)
```

Note the default for the basis dimension in te() is k = 5 and thus lower than in sx(). However, currently **BayesX** does not support a number of inner knots that is lower than 5.

Finally, note that the default for bs in te() is not "ps" but "cr" for cubic regression splines so that te(mbmi, agechild)() cannot be used in bayesx()/bayesx.construct(). Also, te() could in principle set up tensor products of splines with different specifications which is currently not supported in BayesX either and hence results in an error as well.

Markov random fields. The specification of MRFs in sx() and s() is rather straightforward. The main difference is that the list of polygons for the map argument has to be passed to the xt list of extra arguments in s() while it can be supplied directly to sx(). Consequently, the following two specifications lead to identical results when passed to bayesx.construct().

```
R> sx(district, bs = "mrf", map = ZambiaBnd)
R> s(district, bs = "mrf", xt = list(map = ZambiaBnd))
```

D. Additional options for running BayesX

For practical purposes fitting models with function bayesx() is typically sufficient. However, the interfacing functions that are called internally within bayesx() can also be used independently. This could be useful for two reasons: First, users may want to use already existing BayesX program files, and second, there might be a need for automated importing of previously generated BayesX output files into R for further analysis.

Function run.bayesx(), included in package BayesXsrc, is used to run an arbitrary BayesX program file. The arguments of run.bayesx() are

```
run.bayesx(prg = NULL, verbose = TRUE, ...)
```

where prg is a character string with the path to a program file to be executed. If argument prg is not provided BayesX will start in batch mode. During processing of BayesX several pieces of information will be printed to the R console if verbose = TRUE. Further arguments can be passed to function system(), which calls the BayesX binary, using the "..." argument. The function returns a list including the log-file returned by BayesX as well as information on the total runtime.

Model output files are imported using function

```
read.bayesx.output(dir, model.name = NULL)
```

Here, dir is again a directory and model.name is the name of the model the files are imported for, also provided as character strings. Note that the function will search for all different BayesX-estimated models in the declared directory if argument model.name is set to NULL. The returned object is also of class "bayesx", i.e., all the functions and methods described in Table 3 may be applied.

Another noteworthy feature of package R2BayesX is the internal handling of data. BayesX uses numerically efficient algorithms including sparse matrix computations which in principle allow to estimate models using very large datasets. Moreover, the number of different observations for particular covariates is usually much smaller than the total number of observations. That is, the output files returned by the binary only include estimates for unique covariate values. Since these files typically reserve much less disc space, importing the fitted model objects into R using read.bayesx.output() is straightforward in most cases, whereas handling the complete dataset within R may be more burdensome when provided to model fitting functions that do not account for special matrix structures. As mentioned in Section 5.1, users can exploit this by providing a character string to argument data in function bayesx(), which includes the path to a dataset instead of an R data object. As a consequence, this dataset will not be loaded within R and is only used internally by the BayesX binary. To give an example, we generate a large dataset that might produce problems with R's memory allocation using a model fitting function, especially if the model contains a large number of parameters. Therefore, we store the data on disc in the temporary folder of the running session with

```
R> set.seed(321)
R> file <- paste(tempdir(), "/data.raw", sep = "")
R> n <- 5e+06
R> dat <- data.frame(x = rep(runif(1000, -3, 3), length.out = n))
R> dat$y <- with(dat, sin(x) + rnorm(n, sd = 2))
R> write.table(dat, file = file, quote = FALSE, row.names = FALSE)
```

This produces a dataset of approximately 170Mb with only 1000 unique observations for covariate x. The path to the dataset is stored in object file and is provided to argument data in the function call

```
R> b <- bayesx(y ~ sx(x), family = "gaussian", method = "MCMC",
+ iterations = 3000, burnin = 1000, step = 2, predict = FALSE,
+ data = file, seed = 123)</pre>
```

For illustration purposes, the number of iterations is only set to 3000. Note that argument predict is set to FALSE, i.e., only output files of estimated effects will be returned, otherwise an expanded dataset using all observations would be written in the output directory, also containing the data used for estimation. The runtime of this example is about $4 \frac{1}{2}$ hours

```
R> bayesx_runtime(b)
```

```
user system elapsed 16442.12 7.56 16461.33
```

on a Linux system with an Intel 2.33GHz Dual Core processor, while the returned object b uses less than half a megabyte of memory:

```
R> print(object.size(b), units = "Mb")
```

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