



## Estimating Population Abundance Using Sightability Models: R SightabilityModel Package

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### Abstract

Sightability models are binary logistic-regression models used to estimate and adjust for visibility bias in wildlife-population surveys (Steinhorst and Samuel 1989). Estimation proceeds in 2 stages: (1) Sightability trials are conducted with marked individuals, and logistic regression is used to estimate the probability of detection as a function of available covariates (e.g., visual obstruction, group size). (2) The fitted model is used to adjust counts (from future surveys) for animals that were not observed. A modified Horvitz-Thompson estimator is used to estimate abundance: counts of observed animal groups are divided by their inclusion probabilities (determined by plot-level sampling probabilities and the detection probabilities estimated from stage 1). We provide a brief historical account of the approach, clarifying and documenting suggested modifications to the variance estimators originally proposed by Steinhorst and Samuel (1989). We then introduce a new R package, **SightabilityModel**, for estimating abundance using this technique. Lastly, we illustrate the software with a series of examples using data collected from moose (*Alces alces*) in northeastern Minnesota and mountain goats (*Oreamnos americanus*) in Washington State.

*Keywords:* abundance estimation, Horvitz-Thompson, logistic regression, sightability model, R, survey.

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### 1. Introduction

Many wildlife populations are monitored using aerial surveys, in which a random sample of spatial units (or plots) are flown and all observed animals are counted. Typically, not all animals in the selected spatial units will be seen, and thus, population abundance estimators that correct only for sampling will be biased low. Steinhorst and Samuel (1989) developed a method for correcting aerial surveys for undetected animals that is now widely used

to estimate abundance of a variety of wildlife populations, including populations of mountain goats (Rice, Jenkins, and Chang 2009), bighorn sheep (Bodie, Garton, Taylor, and McCoy 1995), Dall’s sheep (Udevitz, Shults, Adams, and Kleckner 2006), moose (Anderson Jr. and Lindzey 1996; Giudice, Fieberg, and Lenarz 2012), elk (Samuel, Garton, Schlegel, and Carson 1987; Otten, Haufler, Winterstein, and Bender 1993; Cogan and Diefenbach 1998; Lubow, Singer, Johnson, and Bowden 2002), burrowing owls (Manning and Garton 76), and various waterfowl species (Pearse, Gerard, Dinsmore, Kaminski, and Reinecke 2008). The approach proceeds in two stages. First, sightability trials are conducted with marked individuals to identify variables (e.g., group size, visual obstruction) that influence the probability that a group of animals will be observed; these data are used to estimate parameters of a logistic regression model. Second, the fitted logistic regression model is used to adjust counts (from future surveys) for animals that were not observed.

Steinhorst and Samuel (1989) suggested a modified Horvitz-Thompson estimator of population size, in which animal counts are divided by their estimated detection probabilities and plot-level sampling probabilities. They derived a complex expression for the variance of the estimator that included 3 separate variance components, that due to: (1) random sampling (of aerial plots); (2) random detection (and missed detection) of independent groups of individuals; and (3) estimation of parameters related to the detection process. Although Steinhorst and Samuel (1989) proposed estimators for these variance components, these estimators were later shown to be biased. Consistent estimators for the variance components were later developed as part of a Ph.D. thesis (Wong 1996), but these estimators have yet to appear in the published literature or in available software. Furthermore, the primary software used to implement the sightability model estimator, program **Aerial Survey** (Unsworth, Leban, Garton, Leptich, and Zager 1999; Leban and Garton 2007), incorrectly implements Steinhorst and Samuel (1989)’s (biased) variance estimator in the case of stratified survey designs (Fieberg and Giudice 2008). Thus, there is a need to develop general software to analyze data from wildlife sightability surveys.

The purpose of this paper is twofold: (1) We aim to provide a historical account of the variance estimators associated with the modified Horvitz-Thompson population abundance estimator originally proposed by Steinhorst and Samuel (1989). (2) We introduce the R package **SightabilityModel** – available from the Comprehensive R Archive Network at <http://CRAN.R-project.org/package=SightabilityModel> – that implements the estimator of Steinhorst and Samuel (1989) along with Wong (1996)’s consistent variance component (and total variance) estimators. We demonstrate the use of the software with data collected from moose (*Alces alces*) in northeastern Minnesota and mountain goats (*Oreamnos americanus*) in Washington State.

## 2. Modified Horvitz-Thompson estimator

### 2.1. Assumptions

The basic assumptions of the sightability model approach, listed below, are taken verbatim from Steinhorst and Samuel (1989):

1. The population is geographically and demographically closed.

2. Groups of animals are independently observed.
3. Observed groups are completely enumerated and observed only once.
4. The survey design for land units can be specified.
5. The probability of observing a group is known or can be estimated.

## 2.2. Notation

Below, we use the notation of [Thompson \(2002\)](#) and [Wong \(1996\)](#). Let:

- $N$  = The number of spatial sampling units (hereafter “plots”) in the population sampling frame.
- $n$  = The number of plots in the sample.
- $I_i$  = Random variable, equal to 1 if the  $i$ -th plot is selected and 0 otherwise.
- $\pi_i$  =  $P(I_i = 1)$ , the probability that the  $i$ -th plot is selected.
- $\pi_{i,i'}$  =  $P(I_i = 1, I_{i'} = 1)$ , the probability that both the  $i$ -th and  $i'$ -th plots are selected.
- $M_i$  = The number of animal groups in the  $i$ -th plot  $\{i = 1, 2, \dots, N\}$ .
- $m_i$  = The number of animal groups observed or sighted in the  $i$ -th sampled plot  $\{i = 1, \dots, n\}$ .
- $y_{i,j}$  = The number of animals in the  $j$ -th group located in the  $i$ -th plot  $\{j = 1, 2, \dots, M_i\}$ .
- $Z_{i,j}$  = Random variable, equal to 1 if the  $j$ -th animal group in the  $i$ -th plot is observed and 0 otherwise.
- $g_{i,j}$  =  $P(Z_{i,j} = 1 | I_i = 1)$ , the probability that the  $j$ -th animal group located in the  $i$ -th sampled plot is observed (conditional on the  $i$ -th plot having been sampled).
- $\theta_{i,j}$  =  $1/g_{i,j}$ , an ‘inflation factor’ associated with the  $j$ -th observed animal group in the  $i$ -th sampled plot.
- $\tau_i$  = The number of animals in the  $i$ -th plot:  $\tau_i = \sum_{j=1}^{M_i} y_{i,j}$ .
- $\tau$  = Total population size:  $\tau = \sum_{i=1}^N \sum_{j=1}^{M_i} y_{i,j} = \sum_{i=1}^N \tau_i$ .

## 2.3. Sightability model approach

The estimator developed in [Steinhorst and Samuel \(1989\)](#) involves the following steps:

1. A set of sightability trials are conducted by flying test plots, each containing a radio-collared animal. Although some animals are involved in multiple test plot surveys, each survey attempt is treated as an independent trial. A suite of animal-specific covariates thought to influence detection probabilities,  $x_i$ , are collected whenever an animal is detected. Similarly, if an animal is not detected when flying a test plot, then telemetry is used to locate the individual immediately after the test plot was surveyed. If the individual is still within the boundaries of the test plot, then the same suite of covariate information is collected and the observation is included in the sightability data frame. The end result is a set of sightability (or detection) observations ( $w_i = 1$  if the radio-collared animal in the  $i$ -th sightability trial was seen and 0 otherwise) and a vector of covariate values ( $x_i$ ) thought to influence detection probabilities.
2. A logistic regression model (relating probability of detection to covariates) is built using data from step 1. Let  $\hat{\beta}$  = estimated regression parameters and  $\hat{\Sigma}$  = estimated variance-covariance matrix of  $\hat{\beta}$ .

3. A new, operational survey, is conducted in which a random sample of plots are surveyed. Within each plot, individuals or groups of individuals are observed, and the same covariates used to model detection probabilities in step 1 are recorded.
4. The regression model from step 2 is used to estimate ‘inflation factors’,  $\hat{\theta}_i =$  inverse probability of detection, for each independent group in step 3; these inflation factors account for animals that were not detected during the operational survey. [Steinhorst and Samuel \(1989\)](#) and [Wong \(1996\)](#) suggest using:

$$\hat{\theta}_i = 1 + \exp(-x_i^\top \hat{\beta} - x_i^\top \hat{\Sigma} x_i / 2) \quad (1)$$

Using large sample maximum likelihood theory and the result that  $\hat{\beta} \xrightarrow{d} N(\beta, \Sigma)$ , [Steinhorst and Samuel \(1989\)](#) showed that this estimator will be consistent for  $\frac{1}{g_{i,j}}$ .

Similarly, [Steinhorst and Samuel \(1989\)](#) suggested the following expressions for  $\widehat{\text{Var}}(\hat{\theta}_i)$  and  $\widehat{\text{Cov}}(\hat{\theta}_i, \hat{\theta}_j)$ , where  $i$  and  $j$  index observations with two different sets of covariates  $x_i$  and  $x_j$ :

$$\widehat{\text{Var}}(\hat{\theta}_i) = \exp(-2x_i^\top \hat{\beta} - 2x_i^\top \hat{\Sigma} x_i) (\exp(x_i^\top \hat{\Sigma} x_i) - 1) \quad (2)$$

$$\widehat{\text{Cov}}(\hat{\theta}_i, \hat{\theta}_j) = \exp(-(x_i + x_j)^\top \hat{\beta} - (x_i + x_j)^\top \hat{\Sigma} (x_i + x_j) / 2) (\exp(x_i^\top \hat{\Sigma} x_j) - 1) \quad (3)$$

These expressions, motivated by the asymptotic normality of  $\hat{\beta}$ , will be needed to derive an expression for  $\widehat{\text{Var}}(\hat{\tau})$ .

5. The total number of animals in the  $i$ -th sampling unit is estimated by summing the number of independently observed animals (or groups of animals) multiplied by their inflation factors (from step 4) across all observed groups in the aerial unit:

$$\hat{\tau}_i = \sum_{j=1}^{M_i} Z_{i,j} y_{i,j} \hat{\theta}_{i,j} = \sum_{j=1}^{m_i} y_{i,j} \hat{\theta}_{i,j}$$

6. The total number of animals in the study area,  $\tau$ , is estimated by dividing values in step 5 by known probabilities of sampling each aerial unit to account for unsampled aerial units:

$$\hat{\tau} = \sum_{i=1}^N \sum_{j=1}^{M_i} \frac{I_i Z_{i,j} y_{i,j} \hat{\theta}_{i,j}}{\pi_i} = \sum_{i=1}^n \sum_{j=1}^{m_i} \frac{y_{i,j} \hat{\theta}_{i,j}}{\pi_i} \quad (4)$$

This estimator is a modified Horvitz-Thompson estimator because it is constructed by dividing each observation by its estimated inclusion probability - i.e., probability of the animal or group of animals being observed during the survey ([Steinhorst and Samuel 1989](#); [Thompson 2002](#)).

Note, the approach can also be applied to situations involving a complete aerial census (by setting all  $\pi_i = 1$ ). In the same way, one can apply the approach to non-randomly sampled plots (but inference should be restricted to these plots). Importantly, Horvitz-Thompson estimators are unbiased if inclusion probabilities are known. When the inclusion probabilities

are not known, the modified Horvitz-Thompson estimator will still be consistent, provided the estimators of the inclusion probabilities are consistent (Thompson 2002).

### 3. Variance estimators

The variance of  $\hat{\tau}$  can be derived by applying the conditional variance formula (Casella and Berger 1990):

$$\text{Var}(Y) = \text{E}_X[\text{Var}(Y|X)] + \text{Var}_X[\text{E}(Y|X)]. \quad (5)$$

To help understand the historical development of variance estimators, it is helpful to distinguish between two cases:

**Case A** Detection (i.e., sightability) probabilities are assumed known.

**Case B** Detection probabilities are estimated.

Following Steinhorst and Samuel (1989) and Wong (1996), we will refer to the variance of  $\hat{\tau}$  as  $\text{Var}(\hat{\tau}_\pi)$  for Case A and  $\text{Var}(\hat{\tau}_{LR})$  for Case B.

#### 3.1. Case A: Detection parameters assumed known

In the case where detection parameters are assumed known, uncertainty arises from random sampling of aerial plots and also from the random detection process, captured by the indicator variables  $I_i$  and  $Z_{i,j}$ , respectively. Conditioning on the  $I_i$ , and applying Equation 5, Steinhorst and Samuel (1989) derived an expression for  $\text{Var}(\hat{\tau}_\pi)$  when detection probabilities are known:

$$\begin{aligned} \text{Var}(\hat{\tau}_\pi) &= \text{Var}_I(\text{E}_{Z|I}(\hat{\tau}_\pi)) + \text{E}(\text{Var}_{Z|I}(\hat{\tau}_\pi)) = V_s + V_d, \text{ with} \\ V_s &= \sum_{i=1}^N \frac{1 - \pi_i}{\pi_i} \tau_i^2 + \sum_{i \neq i'}^N \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_i \pi_{i'}} \tau_i \tau_{i'}. \end{aligned} \quad (6)$$

$$V_d = \sum_{i=1}^N \frac{1}{\pi_i} \sum_{j=1}^{M_i} \frac{1 - g_{i,j}}{g_{i,j}^2} y_{i,j}^2. \quad (7)$$

Thus,  $\text{Var}(\hat{\tau}_\pi)$  is composed of two terms (or components), the first of which is due to sampling ( $V_s$ ); the second is due to the random detection process ( $V_d$ ). Steinhorst and Samuel (1989) suggested estimators for these variance components, which we will label  $v_s$  and  $v_d$ :

$$v_s = \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \hat{\tau}_i^2 + \sum_{i \neq i'}^n \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_{i,i'} \pi_i \pi_{i'}} \hat{\tau}_i \hat{\tau}_{i'} \quad (8)$$

$$v_d = \sum_{i=1}^n \frac{1}{\pi_i^2} \sum_{j=1}^{m_i} \frac{1 - g_{i,j}}{g_{i,j}^2} y_{i,j}^2, \quad (9)$$

where  $\hat{\tau}_i = \sum_{j=1}^{m_i} y_{i,j} \hat{\theta}_{i,j}$ .

Thompson and Seber (1996) also derived an expression for  $\text{Var}(\hat{\tau}_\pi)$  using Equation 5, but in contrast to Steinhorst and Samuel (1989), they derived their expression by conditioning on

the detection indicators,  $Z_{i,j}$ :

$$\text{Var}(\hat{\tau}_\pi) = \mathbb{E}_Z(\text{Var}_{I|Z}(\hat{\tau}_\pi)) + \text{Var}_Z(\mathbb{E}_{I|Z}(\hat{\tau}_\pi)) = V_1 + V_2, \text{ with}$$

$$V_1 = \mathbb{E}_Z\left(\sum_{i=1}^N \frac{1 - \pi_i}{\pi_i} \hat{\tau}_i^2 + \sum_{i \neq i'}^N \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_i \pi_{i'}} \hat{\tau}_i \hat{\tau}_{i'}\right) \quad (10)$$

$$V_2 = \sum_{i=1}^N \sum_{j=1}^{M_i} \frac{1 - g_{i,j}}{g_{i,j}} y_{i,j}^2. \quad (11)$$

Thompson and Seber (1996) did not evaluate the expectation in the expression for  $V_1$  (Equation 10). Nonetheless, they recognized that the two variance decompositions give equivalent expressions for  $\text{Var}(\hat{\tau})$  (i.e.,  $V_s + V_d = V_1 + V_2$ ). Using this equivalence, we can solve for  $V_1$ :

$$V_1 = \sum_{i=1}^N \frac{1 - \pi_i}{\pi_i} \tau_i^2 + \sum_{i \neq i'}^N \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_i \pi_{i'}} \tau_i \tau_{i'} + \sum_{i=1}^N \frac{1 - \pi_i}{\pi_i} \sum_{j=1}^{M_i} \frac{1 - g_{i,j}}{g_{i,j}} y_{i,j}^2. \quad (12)$$

Thompson and Seber (1996) also showed that  $v_d$  was an unbiased estimator for  $V_d$ , but that  $v_s$  was a biased estimator of  $V_s$ . Lastly, Thompson and Seber (1996) derived unbiased estimators ( $v_1$  and  $v_2$ ) for the two terms in their variance decomposition and, thus, provided an unbiased estimator for  $\text{Var}(\hat{\tau}_\pi)$  when detection probabilities are assumed known:

$$v_1 = \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \hat{\tau}_i^2 + \sum_{i \neq i'}^n \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_{i,i'} \pi_i \pi_{i'}} \hat{\tau}_i \hat{\tau}_{i'} \quad (13)$$

$$v_2 = \sum_{i=1}^n \frac{1}{\pi_i} \sum_{j=1}^{m_i} \frac{1 - g_{i,j}}{g_{i,j}^2} y_{i,j}^2. \quad (14)$$

Note that  $v_s$  is the same as  $v_1$ , but  $v_d$  and  $v_2$  differ by a factor of  $1/\pi_i$ . Lastly, noting that Steinhorst and Samuel (1989)'s estimator,  $\widehat{\text{Var}}(\hat{\tau}_\pi) = v_s + v_d$  is equal to  $v_1 + v_d$ , Thompson and Seber (1996) were able to derive an unbiased estimate of  $V_s$ , which we will label  $v'_s$ :

$$\begin{aligned} v'_s &= v_1 + v_2 - v_d \\ &= \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \hat{\tau}_i^2 + \sum_{i \neq i'}^n \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_{i,i'} \pi_i \pi_{i'}} \hat{\tau}_i \hat{\tau}_{i'} - \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \sum_{j=1}^{m_i} \frac{1 - g_{i,j}}{g_{i,j}^2} y_{i,j}^2 \end{aligned} \quad (15)$$

Wong (1996) derived the same estimator (i.e.,  $v'_s$  for  $V_s$ ) in her Ph.D. dissertation.

Interestingly, Samuel, Steinhorst, Garton, and Unsworth (1992) developed sightability estimators for additional population quantities (e.g., sex ratios), and in their Appendix, they suggested a new variance component estimator for  $V_d$  which turns out to be  $v_2$ . Thus, Samuel *et al.* (1992)'s suggestion results in *biased* estimators for each individual variance component, but an *unbiased* estimate of  $\text{Var}(\hat{\tau}_\pi)$ , equivalent to Thompson and Seber (1996)'s estimator  $\widehat{\text{Var}}(\hat{\tau}_\pi) = v_1 + v_2$ , if detection probabilities are assumed known.

### 3.2. Case B: Detection parameters are estimated

Steinhorst and Samuel (1989) derived an expression for  $\text{Var}(\hat{\tau}_{LR})$ , by first noting that:

$$\text{Var}(\hat{\tau}_{LR}) = \text{Var}(\hat{\tau}_\pi) + \mathbb{E}_{Z,I} \text{Var}(\hat{\tau}_{LR} | Z, I), \quad (16)$$

where again,  $\text{Var}(\hat{\tau}_\pi)$  denotes the variance for the case where detection parameters are assumed to be known. This last term,  $\text{E}_{Z,I}\text{Var}(\hat{\tau}_{LR}|Z, I)$ , captures uncertainty in  $\hat{\tau}_{LR}$  associated with the estimated (logistic regression) detection parameters. Thus, [Steinhorst and Samuel \(1989\)](#) expressed the total variance in terms of 3 components, sampling [ $V_s$ ], sightability [ $V_d$ ], and that due to modeling the detection probabilities [ $V_m$ ]. [Steinhorst and Samuel \(1989\)](#) further showed that:

$$\begin{aligned} V_m &= \text{E}_{Z,I}\text{Var}(\hat{\tau}_{LR}|Z, I) \\ &= \sum_{i=1}^N \frac{1}{\pi_i} \sum_{j=1}^{M_i} \frac{y_{i,j}^2}{\theta_{i,j}} \text{Var}(\hat{\theta}_{i,j}) + \sum_{i=1}^N \frac{1}{\pi_i} \sum_{j \neq j'}^{M_i} \frac{y_{i,j} y_{i,j'}}{\theta_{i,j} \theta_{i,j'}} \text{Cov}(\hat{\theta}_{i,j}, \hat{\theta}_{i,j'}) \\ &\quad + \sum_{i \neq i'}^N \frac{\pi_{i,i'}}{\pi_i \pi_{i'}} \sum_{j=1}^{M_i} \sum_{j'=1}^{M_{i'}} \frac{y_{i,j} y_{i',j'}}{\theta_{i,j} \theta_{i',j'}} \text{Cov}(\hat{\theta}_{i,j}, \hat{\theta}_{i',j'}) \end{aligned} \quad (17)$$

[Steinhorst and Samuel \(1989\)](#) and [Wong \(1996\)](#) derived the same estimator for  $V_m$ , which we will refer to as  $v_m$ :

$$\begin{aligned} v_m &= \sum_{i=1}^n \frac{1}{\pi_i^2} \sum_{j=1}^{m_i} y_{i,j}^2 \widehat{\text{Var}}(\hat{\theta}_{i,j}) + \sum_{i=1}^n \frac{1}{\pi_i^2} \sum_{j \neq j'}^{m_i} y_{i,j} y_{i,j'} \widehat{\text{Cov}}(\hat{\theta}_{i,j}, \hat{\theta}_{i,j'}) \\ &\quad + \sum_{i \neq i'}^n \frac{1}{\pi_i \pi_{i'}} \sum_{j=1}^{m_i} \sum_{j'=1}^{m_{i'}} y_{i,j} y_{i',j'} \widehat{\text{Cov}}(\hat{\theta}_{i,j}, \hat{\theta}_{i',j'}), \end{aligned} \quad (18)$$

where  $\widehat{\text{Var}}(\hat{\theta}_{i,j})$  and  $\widehat{\text{Cov}}(\hat{\theta}_{i,j}, \hat{\theta}_{i',j'})$  are given by Equation 2 and Equation 3.

[Steinhorst and Samuel \(1989\)](#) suggested estimating  $\text{Var}(\hat{\tau}_{LR})$  by combining  $v_m$  with  $v_s$  (Equation 8) and  $v_d$  (Equation 9). [Wong \(1996\)](#) showed that while  $v_m$  was a consistent estimator of  $V_m$ ,  $v_s$  and  $v_d$  were biased (away from 0) when detection parameters were estimated. She derived the following consistent estimators of  $V_s$  and  $V_d$ ,  $v_s^*$  and  $v_d^*$  respectively, for the case where detection probabilities are estimated using logistic regression:

$$\begin{aligned} v_s^* &= \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \hat{\tau}_i^2 + \sum_{i \neq i'}^n \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_{i,i'} \pi_i \pi_{i'}} \hat{\tau}_i \hat{\tau}_{i'} - \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \sum_{j=1}^{m_i} y_{i,j}^2 (\hat{\theta}_{i,j}^2 - \hat{\theta}_{i,j}) \\ &\quad - \sum_{i=1}^n \frac{1 - \pi_i}{\pi_i^2} \sum_{j \neq j'}^{m_i} y_{i,j} y_{i,j'} \widehat{\text{Cov}}(\hat{\theta}_{i,j}, \hat{\theta}_{i,j'}) \\ &\quad - \sum_{i \neq i'}^n \frac{\pi_{i,i'} - \pi_i \pi_{i'}}{\pi_{i,i'} \pi_i \pi_{i'}} \sum_{j=1}^{m_i} \sum_{j'=1}^{m_{i'}} y_{i,j} y_{i',j'} \widehat{\text{Cov}}(\hat{\theta}_{i,j}, \hat{\theta}_{i',j'}) \end{aligned} \quad (19)$$

$$v_d^* = \sum_{i=1}^n \frac{1}{\pi_i^2} \sum_{j=1}^{m_i} y_{i,j}^2 (\hat{\theta}_{i,j}^2 - \hat{\theta}_{i,j} - \widehat{\text{Var}}(\hat{\theta}_{i,j})) \quad (20)$$

Although these estimators have not been published in peer-reviewed literature, they were used by [Lubow et al. \(2002\)](#) to analyze data collected on elk (*Cervus elaphus*) data in Rocky Mountain National park (Colorado, USA).

### 3.3. Confidence intervals

The sampling distribution of  $\hat{\tau}$  is often skewed right, and as a result, confidence intervals constructed under an asymptotic normality assumption often fail to have correct coverage

rates (Wong 1996; Cogan and Diefenbach 1998). Using simulated data, Wong (1996) explored several alternative methods for constructing confidence intervals. Although the relative performance of the various methods depended on the specifics of the simulation, intervals constructed under the assumption that  $(\hat{\tau} - T)$  is lognormally distributed, where  $T$  is the total number of animals seen, tended to perform well across a range of simulation scenarios. Using this approach, confidence limits are formed using:

$$LCL = T + [(\hat{\tau} - T)/C]\sqrt{(1 + cv^2)} \quad (21)$$

$$UCL = T + [(\hat{\tau} - T)C]\sqrt{(1 + cv^2)}, \quad (22)$$

where  $LCL$  and  $UCL$  are the lower and upper confidence limits, respectively,  $cv^2 = \text{var}(\hat{\tau})/(\hat{\tau} - T)^2$ , and  $C = \exp[z_{\alpha/2}\sqrt{\ln(1 + cv^2)}]$ .

## 4. Package description and example applications

The **SightabilityModel** package implements Steinhorst and Samuel (1989)'s logistic regression sightability abundance estimator via the `Sight.Est` function. This function requires, as arguments, the original sightability trial data frame and an operational survey data frame. Alternatively, one can specify a pre-fitted model (using  $\hat{\beta}$  and  $\hat{\Sigma}$  as arguments to `Sight.Est`). Covariates used in the sightability model must be present in both data frames (with identical naming conventions in both). The operational data frame also requires variables named `total` (containing the animal counts,  $y_{i,j}$ ) and `subunit` (a sample plot identifier). Most operational surveys employ stratified random sampling designs to select survey plots (as a means of increasing precision), with plots allocated to strata based on their expected animal densities (e.g., 'low', 'medium', and 'high'; Fieberg and Lenarz 2012). Thus, the operational data frame must also include a variable named `stratum`, which serves as a stratum identifier (`stratum` should take on a single value for non-stratified surveys). Lastly, the user must supply a data frame containing sampling information, including `nh` (number of sampled plots in each stratum), `Nh` (number of population units in each stratum), and a variable named `stratum` (taking on the same values as the `stratum` variable in the operational data frame); for non-stratified survey designs this data frame will contain a single record.

The `Sight.Est` function fits a logistic regression model to the sightability data frame and applies the model to data from the operational survey to estimate population abundance,  $\hat{\tau}$ . The variance of  $\hat{\tau}$  can be estimated using  $\widehat{\text{Var}}(\hat{\tau}_{LR}) = v_1 + v_2 + v_m$  as suggested by Samuel *et al.* (1992) or using Wong (1996)'s estimator  $\widehat{\text{Var}}(\hat{\tau}_{LR}) = v_s^* + v_d^* + v_m$  (recommended). Alternatively, one can use a nonparametric bootstrap to aid in the estimation of the variance components (see Example 5 in the next section).

### 4.1. Example 1: Application to moose survey data

We illustrate the software using data collected from moose (*Alces alces*) in northeastern Minnesota (Giudice *et al.* 2012). Sightability trial data were collected from 2005–2007 (124 trials were conducted and the results are captured in the `exp.m` data frame); operational survey data from 2004–2007 are also included in the package (`obs.m` data frame). Lastly, sampling information from the 2004–2007 operational surveys is captured in the `sampinfo.m` data frame.

```
R> library("SightabilityModel")
R> data("obs.m")
R> data("exp.m")
R> data("sampinfo.m")
R> exp.m[1:5, ]
```

	year	observed	voc	grpsize
37	2005	1	20	4
38	2005	1	85	2
39	2005	0	80	1
40	2005	0	75	1
41	2005	0	70	1

The experimental data frame contains 4 variables: **year** (year of the test trial), **observed** (equal to 1 if the moose was observed and 0 otherwise), **voc** (amount of screening cover within 4 animal lengths of the first animal seen), and **grpsize** (number of animals associated with the radiocollared animal, i.e., cluster size). Each row represents an independent sightability trial, with **observed** representing the random variables ( $w_i$ ).

```
R> obs.m[1:5, ]
```

	year	stratum	subunit	total	cows	calves	bulls	unclass	voc	grpsize
1	2004	1	140	2	1	1	0	0	90	2
2	2004	1	140	1	0	0	1	0	90	1
3	2004	1	140	2	0	0	2	0	75	2
4	2004	1	140	3	1	0	2	0	0	3
5	2004	1	180	2	2	0	0	0	85	2

Each record in the operational data frame corresponds to an independently sighted group of moose, with variables that capture animal-specific covariates,  $x_i = (\text{voc}, \text{grpsize})$ , for potential inclusion in the detection model. The data frame also contains plot-level information (**stratum** = stratum identifier, **subunit** = sample plot identifier). Lastly, **total** gives the total number of animals observed in each independently sighted group (note: this variable is redundant with **grpsize**; the latter variable is included to allow one to model detection probabilities as a function of group size).

Lastly, the **sampinfo.m** data frame contains sampling information associated with the operational surveys conducted in years 2004–2007. Specifically, it contains the number of sampled ( $n_h$ ) and population ( $N_h$ ) units in each stratum:

```
R> sampinfo.m
```

	year	stratum	Nh	nh
1	2004	1	178	10
2	2004	2	127	14
3	2004	3	31	6
4	2005	1	238	12

```

5 2005      2 180 18
6 2005      3  35  6
7 2006      1 238 18
8 2006      2 180 14
9 2006      3  35  5
10 2007     1 248 16
11 2007     2 170 18
12 2007     3  35  6

```

The `Sight.Est` function is used both to fit a specified logistic regression model and estimate population abundance in a single step. Below, we illustrate the code using operational survey data collected in 2004 only, modeling detection probabilities as a function of visual obstruction (i.e., `voc`; note, an intercept-only model, in which detection probabilities are assumed to be constant, can be specified using `observed~1` in the `Sight.Est` function):

```
R> est.2004 <- Sight.Est(observed ~ voc, odat = subset(obs.m, year == 2004),
+   sdat = exp.m, sampinfo = subset(sampinfo.m, year == 2004))
```

By default, [Wong \(1996\)](#)'s estimators are used to estimate  $\text{Var}(\hat{\tau}_{LR})$ , and confidence intervals are formed under the assumption that  $(\hat{\tau} - T)$  is lognormally distributed. The print function provides information on the fitted sightability model and sampling statistics. In addition, it provides the point estimate and confidence interval as well as estimates of each of the 3 variance components.

```
R> print(est.2004)
```

Call:

```
Sight.Est(form = observed ~ voc, sdat = exp.m, odat = subset(obs.m,
  year == 2004), sampinfo = subset(sampinfo.m, year == 2004))
```

```
----- SIGHTABILITY MODEL -----
```

```
Call:  glm(formula = form, family = binomial(), data = sdat)
```

Coefficients:

```
(Intercept)      voc
    1.75993      -0.03479
```

```
Degrees of Freedom: 123 Total (i.e. Null); 122 Residual
```

```
Null Deviance:      171.6
```

```
Residual Deviance: 147.4    AIC: 151.4
```

```
----- Population Survey data -----
```

Stratum Sampling Information

```
  stratum nh  Nh
1         1 10 178
```

```

2      2 14 127
3      3  6  31

```

```

Number of animals seen in each stratum

```

```

  1  2  3
68 248 202

```

```

----- POPULATION ESTIMATE ( 95 % CI) -----

```

```

[1] tau.hat = 13,096 ; 95 % CI = ( 8,549 , 21,430 )

```

```

----- SE(tau.hat) -----

```

```

Variance method: [1] "WONG"

```

```

  SE
3,117

```

```

----- Variance Components -----

```

```

  VarSamp  VarSight  VarMod
3,033,952  873,246 5,810,441

```

Note: the variance component estimates provide useful information for improving future surveys. In particular, the first and third variance components (sampling uncertainty and parameter uncertainty associated with the detection model, respectively) are under control of the observer. Sampling uncertainty can be reduced by surveying a larger number of aerial plots or by implementing a more efficient sampling design (Fieberg and Lenarz 2012), whereas parameter uncertainty can be reduced by conducting more sightability trials. The large variance component associated with model parameters (i.e., `VarMod`), suggests that conducting additional sightability trials would be useful, particularly since the benefits would be realized in multiple years (i.e., all estimates using the same sightability model will be improved by reducing this variance component).

For a more concise summary, one can use the `summary` function, which returns the point estimate and confidence interval:

```

[1] tau.hat = 13,096 ; 95 % CI = ( 8,549 , 21,430 )

```

#### 4.2. Example 2a: Correlated stratum-specific estimates

Stratified random sampling is often used to select plots in aerial surveys. Unlike traditional survey estimates that employ stratified random sampling, stratum-specific abundance estimates will be correlated when a common detection model is used to correct counts in all strata (Fieberg and Giudice 2008). Unfortunately, many authors have mistakenly assumed stratum-specific variances will sum to give the total variance; ignoring the correlation among stratum-specific estimates will typically result in an underestimate of the total variance (Fieberg and Giudice 2008). This next example highlights this point.

We begin by analyzing 2004 survey data for each stratum separately, storing the results in a matrix named `tau.hats`:

```
R> tau.hats <- matrix(NA, 3, 5)
R> rownames(tau.hats) <- c("Stratum 1", "Stratum 2", "Stratum 3")
R> for(i in 1:3) {
+   tempsamp <- sampinfo.m[i, ]
+   tempobs <- obs.m[obs.m$year == 2004 & obs.m$stratum == i, ]
+   temp <- Sight.Est(observed ~ voc, odat = tempobs, sdat = exp.m,
+     sampinfo = tempsamp)
+   tau.hats[i, ] <- temp$est
+ }
R> colnames(tau.hats) <- names(temp$est)
R> tau.hats <- round(tau.hats, 0)
R> print(format(tau.hats, big.mark = ","), quote = FALSE)
```

	tau.hat	VarTot	VarSamp	VarSight	VarMod
Stratum 1	4,152	2,892,280	1,555,085	487,767	849,429
Stratum 2	5,098	1,389,226	797,124	191,047	401,055
Stratum 3	3,846	1,679,580	681,744	194,431	803,405

We then compare the sum of the stratum-specific abundance estimates and variance component estimates to those obtained by applying the `Sight.Est` function once (i.e., to data from all strata).

```
R> est.2004 <- Sight.Est(observed ~ voc, odat = subset(obs.m, year == 2004),
+   sdat = exp.m, sampinfo = subset(sampinfo.m, year == 2004))
R> print(format(round(est.2004$est, 0), big.mark = ","), quote = FALSE)
```

tau.hat	VarTot	VarSamp	VarSight	VarMod
13,096	9,717,638	3,033,952	873,246	5,810,441

```
R> naive.tau.hats <- round(apply(tau.hats[1:3, ], 2, sum), 0)
R> print(format(naive.tau.hats, big.mark = ","), quote = FALSE)
```

tau.hat	VarTot	VarSamp	VarSight	VarMod
13,096	5,961,086	3,033,953	873,245	2,053,889

Note that the point estimate formed by summing the stratum-specific abundance estimates is correct, but the sum of the stratum-specific variances is too small (5,961,086 compared to 9,717,638). The difference is due to the positive covariance among the stratum-specific estimates as a result of applying the same detection model to each stratum (note the variance components due to sampling and sightability are independent across strata).

Thus, if stratum-specific estimates are of interest, they can be obtained using separate calls to `Sight.est`, but the variance estimate for the total population size (i.e., aggregated across strata) requires that all data be processed simultaneously (e.g., using an additional call to `Sight.Est` with all of the operational survey data included).

### 4.3. Example 2b: Non-independent population estimates

Often, management agencies are interested in changes in population size (e.g., from one year to the next). Similar to stratum-specific estimates, population estimates will not be independent across years if they are formed using the same sightability model. Typically, estimates will exhibit a positive covariance, making tests (for a difference between years) conservative if estimates are assumed to be independent.

The `vardiff` function can be used to estimate the variance of a difference between two population estimates, while accounting for any covariance between the estimates due to using the same sightability model. The function takes as arguments 2 sightability model objects created from separate calls to the `Sight.Est` function.

```
R> est.2006 <- Sight.Est(observed ~ voc, odat = subset(obs.m, year == 2006),
+   sdat = exp.m, subset(sampinfo.m, year == 2006))
R> est.2007 <- Sight.Est(observed ~ voc, odat = subset(obs.m, year == 2007),
+   sdat = exp.m, subset(sampinfo.m, year == 2007))
R> vdiff<-vardiff(est.2006, est.2007)
R> print(format(vdiff, nsmall = 0, big.mark = ","), quote = FALSE)
```

```
      Var(difference)
[1,] 2,562,188
```

```
R> naive <- est.2006$est[2] + est.2007$est[2]
R> print(format(naive, nsmall = 0, big.mark = ","), quote = FALSE)
```

```
      VarTot
3,649,057
```

As expected, the estimates of moose abundance in 2006 and 2007 exhibit a positive covariance due to using the same sightability model to correct for detection. Thus, the naive estimate (formed by summing the individual variances as though the two estimates were independent) is too large. We therefore recommend using the `vardiff` function whenever estimating differences in population size across years. We also provide a function (`varlog.lam`) to calculate the variance of the log rate of change between two population estimates (i.e.,  $var(\log(\hat{\tau}_2/\hat{\tau}_1))$ ). Lastly, it is interesting to note that naive estimates of variance (assuming independence) were too small in Example 2a, but too large in Example 2b. In both cases, individual estimates were positively correlated because they were formed using the same sightability model. These results follow from the fact that Example 1a involves estimation of a sum [with  $var(x + y) = var(x) + var(y) + 2cov(x, y)$ ] whereas Example 1b involves estimation of a difference [with  $var(x - y) = var(x) + var(y) - 2cov(x, y)$ ].

### 4.4. Example 3: Non-linear detection functions

Giudice *et al.* (2012) considered models in which the logit detection probability varied non-linearly as a function of `voc`. In this next example, we show how the `ns` function in the `splines` package of Program R (R Development Core Team 2012) can be used to allow for non-linear sightability models (on the logit scale). To do so, we need to create the spline basis functions

(in both experimental and operational data frames) first before calling `Sight.Est`. Below, we allocate 3 degrees of freedom to model the effect of `voc` on the probability of detection.

```
R> library("splines")
```

```
R> exp.m$voc.ns <- ns(exp.m$voc, df = 3)
```

```
R> obs.m$voc.ns <- predict(exp.m$voc.ns, obs.m$voc)
```

```
R> ns.est <- Sight.Est(observed ~ voc.ns, odat = subset(obs.m, year == 2004),
+   sdat = exp.m, subset(sampinfo.m, year == 2004))
```

```
R> ns.est$sight
```

```
Call: glm(formula = form, family = binomial(), data = sdat)
```

```
Coefficients:
```

```
(Intercept)      voc.ns1      voc.ns2      voc.ns3
      2.858      -2.056      -6.902      -2.565
```

```
Degrees of Freedom: 123 Total (i.e. Null); 120 Residual
```

```
Null Deviance:      171.6
```

```
Residual Deviance: 145.7    AIC: 153.7
```

```
R> print(format(round(ns.est$est, 0), big.mark = ","), quote = FALSE)
```

```
tau.hat      VarTot      VarSamp      VarSight      VarMod
12,585 10,744,879 2,559,523    608,521 7,576,835
```

#### 4.5. Example 4: Using multi-model inference techniques

Many sightability studies collect a wide array of covariate data for relatively few numbers of sightability trials. Developing predictive models that perform well on future data can be challenging in these situations (Giudice *et al.* 2012). Model averaging can serve as a form of shrinkage, and thus may help to improve predictive accuracy when applying the model to new data (Burnham and Anderson 2002; Giudice *et al.* 2012). Burnham and Anderson (2002) describe a popular approach for performing model averaging in wildlife research, including the calculation of unconditional variance-covariance matrices that attempt to account for model selection uncertainty.

Model-averaged parameter estimates and unconditional variance-covariance matrices can be used to estimate abundance, by passing these values as arguments to the `Sight.Est` function. For illustrative purposes, we consider multi-model inference results from Rice *et al.* (2009), applied to an operational survey of mountain goats in Olympic National Park, Washington. Model-averaged regression parameters (for covariates `GroupSize`, `Terrain`, and `pct.VegCover`), and the corresponding unconditional variance-covariance matrix from Rice *et al.* (2009) are stored in a list named `g.fit`:

```
R> data("g.fit")
```

```
R> g.fit
```

```

$beta.g
[1] 1.385 0.158 -1.138 -0.012

$varbeta.g
      [,1]      [,2]      [,3]      [,4]
[1,] 0.160801 -0.013000 -0.090000 -1.0e-03
[2,] -0.013000 0.003721 -0.001000 0.0e+00
[3,] -0.090000 -0.001000 0.161604 0.0e+00
[4,] -0.001000 0.000000 0.000000 8.1e-05

```

The operational survey data are contained in a data frame named `gdat`.

```

R> data("gdat")
R> gdat[1:5, ]

```

	GroupSize	Terrain	pct.VegCover	stratum	total	subunit
1	1	0	38.00	1	1	69
2	1	0	38.00	1	1	69
3	1	0	3.25	1	1	69
4	1	0	3.25	1	1	69
5	1	0	1.77	1	1	69

We create a data frame to hold the sampling information for the operational survey:

```

R> sampinfo <- data.frame(nh = c(6, 23, 11), Nh = c(6, 27, 65),
+   stratum = c(1, 2, 3))

```

Although 3 strata were surveyed, no mountain goats were observed in the 11 low stratum plots.

```

R> table(gdat$stratum)

```

```

 1  2
50 27

```

Thus, we only supply the first two records of `sampinfo` when we call `Sight.Est`. We also need to specify `bet=beta.g` and `varbet=varbeta.g` to indicate that we are supplying our own regression model (fit outside of `Sight.Est`):

```

R> goat.est <- Sight.Est(observed ~ GroupSize + Terrain + pct.VegCover,
+   odat = gdat, sampinfo = sampinfo[1:2, ], bet = g.fit$beta.g,
+   varbet = g.fit$varbeta.g)
R> print(goat.est)

```

Call:

```

Sight.Est(form = observed ~ GroupSize + Terrain + pct.VegCover,
  odat = gdat, sampinfo = sampinfo[1:2, ], bet = g.fit$beta.g,

```

```

varbet = g.fit$varbeta.g)

----- SIGHTABILITY MODEL -----
$bet
[1] 1.385 0.158 -1.138 -0.012

$varbet
      [,1]      [,2]      [,3]      [,4]
[1,] 0.160801 -0.013000 -0.090000 -1.0e-03
[2,] -0.013000 0.003721 -0.001000 0.0e+00
[3,] -0.090000 -0.001000 0.161604 0.0e+00
[4,] -0.001000 0.000000 0.000000 8.1e-05

$note
[1] "User supplied regression model"

----- Population Survey data -----

Stratum Sampling Information
  stratum nh Nh
1         1 6 6
2         2 23 27

Number of animals seen in each stratum
  1 2
124 56

----- POPULATION ESTIMATE ( 95 % CI) -----

[1] tau.hat = 230 ; 95 % CI = ( 205 , 293 )

----- SE(tau.hat) -----
Variance method: [1] "WONG"
SE
20

----- Variance Components -----

VarSamp VarSight VarMod
      137      168      82

```

#### 4.6. Example 5: Use of the bootstrap to estimate variance components

Fieberg and Giudice (2008) used simulations to explore the reliability of the estimators of

$\text{Var}(\theta_{i,j})$  and  $\text{Cov}(\theta_{i,j}, \theta_{i',j'})$  given in Equation 2 and 3. They found that these variance and covariance terms were generally underestimated when a small number of sightability trials were used to estimate  $\beta$  (e.g.,  $< 100$ ), and suggested a non-parametric bootstrap (e.g., resampling sightability data with replacement) might be useful for estimating these terms (see also e.g., Wong 1996; Cogan and Diefenbach 1998).

If the user specifies `Vm.boot = TRUE` when calling the `Sight.Est` function,  $\widehat{\text{Var}}(\theta_{i,j})$  and  $\widehat{\text{Cov}}(\theta_{i,j}, \theta_{i',j'})$  will be estimated by applying a non-parametric bootstrap to the sightability data frame. Specifically, the logistic regression model is fit to `nboot` bootstrap data sets and  $\text{Var}(\theta_{i,j})$  and  $\text{Cov}(\theta_{i,j}, \theta_{i',j'})$  are estimated using empirical variance/covariances across bootstrap replicates. These estimates are then plugged into formulas used for estimating  $V_s, V_d$ , and  $V_m$ .

In this example, we compare analytical and bootstrap estimates of variance (with 10,000 bootstrap replicates), considering only the subset of moose experimental data collected in 2005 (representing a total of 39 sightability trials).

```
R> analytical.est <- Sight.Est(observed ~ voc,
+   odat = subset(obs.m, year == 2004),
+   sdat = subset(exp.m, year == 2005), subset(sampinfo.m, year == 2004),
+   method = "Wong", logCI = TRUE, alpha = 0.05, Vm.boot = FALSE)
R> print(format(round(analytical.est$est, 0), big.mark = ","),
+   quote = FALSE)
```

tau.hat	VarTot	VarSamp	VarSight	VarMod
8,289	3,266,683	1,184,332	183,050	1,899,301

```
R> boot.est <- Sight.Est(observed ~ voc, odat = subset(obs.m,
+   year == 2004), sdat=subset(exp.m, year == 2005), subset(sampinfo.m,
+   year == 2004), method = "Wong", logCI = T, alpha = 0.05,
+   Vm.boot = TRUE, nboot = 10000)
R> print(format(round(boot.est$est, 0), big.mark = ","), quote = FALSE)
```

tau.hat	VarTot	VarSamp	VarSight	VarMod
8,289	5,598,611	1,117,145	146,152	4,335,314

As expected, the variance estimate calculated using the bootstrap is larger than the analytical estimate (constructed using the estimators in Equation 2 and Equation 3).

## 5. Future work

Another potential concern with applying the sightability model approach is that group size, a key covariate in many detection models, is often estimated with error (Cogan and Diefenbach 1998; Walsh, Page, Campa, Winterstein, and Beyer 2009). In addition to problems associated with estimating regression parameters when covariates are subject to measurement error, animal counts,  $y_{i,j}$ , will usually be too small. Walsh *et al.* (2009) suggested a double observer approach to correct for this problem. We hope to incorporate this option in a future version of the **SightabilityModel** package. In addition, we are currently exploring Bayesian

implementations that utilize data augmentation, similar to methods developed for analyzing mark-recapture data with individual covariates (Royle 2009; Royle and Dorazio 2012). Lastly, we hope to incorporate sightability estimators of other population quantities (e.g., calf:cow ratios) as described by Samuel *et al.* (1992).

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