Using Mathematica to build Non-parametric Statistical Tables

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Abstract

In this paper, I present computational procedures to obtain statistical tables. The tables of the asymptotic distribution and the exact distribution of Kolmogorov-Smirnov statistic D_n for one population, the table of the exact distribution of Kolmogorov-Smirnov statistic $D_{m,n}$ for two populations, the table of the distribution of the runs R, the table of the distribution of Wilcoxon signed-rank statistic W^+ and the table of the distribution of Mann-Whitney statistic U_x using Mathematica, Version 3.0 under Windows98. I think that it is an interesting cuestion because many statistical packages give the asymptotic significance level in the statistical tests and with these procedures one can easily calculate the exact significance levels and the left-tail and right-tail probabilities with non-parametric distributions.

I have used mathematica to make theses calculations because one can use symbolic language to solve recursion relations. It's very easy to generate the format of the tables, and it's possible to obtain any table of the mentioned non-parametric distributions with any precision, not only with the standard parameters more used in Statistics, and without transcription mistakes.

Furthermore, using similar procedures, we can generate tables for the following distribution functions: Binomial, Poisson, Hypergeometric, Normal, χ^2 Chi-Square, T-Student, F-Snedecor, Geometric, Gamma o Beta.

Key Words: Mathematica, nonparametric distributions, tables.
AMS: 62Q05.

1 Introduction

This work is divided in two sections. In the first section I describe the procedures and the code to generate the tables and in the second one I present the computational results i.e. the tables.

To build the tables of the mentioned distributions I have used the commands Do[] to repeat calculations, Sum[] to sum series, FindRoot[] to solve equations, Flatten[] to join list of values, GrayCode[] to generate subsets of the list of values generated with Range[], Extract[] to extract elements of the list, IntegerPart[] to calculate the integer part, Join[] to join lists, Frequencies[] to calculate the frequencies distribution and some commands to do the tables and style specifications as Table[] to generate a table of values, Prepend[], TableForm[], Print[] and StyleForm[].

To calculate the tables of the following distributions: Binomial, Poisson, Hypergeometric, Normal, χ^2 Chi-square, T-Student, F-Snedecor, Geometric, Gamma or Beta the essential commands are PDF[] (Probability Distribution Function) and CDF[] (Cumulative Distribution Function).

2 Procedures to obtain tables.

2.1 Table of the asymptotic distribution of statistic D_n

This table is used in the goodness-of-fit Kolmogorov-Smirnov test with sample size $n > 40$,

$$\begin{cases} H_0 : F(x) = F_0(x) \ \forall x \in \Re. \\ H_1 : F(x) \neq F_0(x) \ for \ some \ x \in \Re. \end{cases}$$

The Kolmogorov-Smirnov test is designed for continuous distributions F_0. The sample have not been grouped in categories as in the chi-square goodness of fit test.

The Kolmogorov-Smirnov one-sample statistic is based on the differences between the hypothesized cumulative distribution function $F_0(x)$ and the empirical distribution function of the sample $F_n(x) = \frac{N(x)}{n}$, where $N(x)$ is the number of sample values less than or equal to x.

The deviations between the true distribution function and its statistical image should be small for all values of x and we can use the statistic $D_n = max_x|F_n(x) - F_0(x)|$ to measure the accuracy of our estimate. Given the significance level α, the critical region is $D_n > D_{n,\alpha}$.

This table it is also used in the kolmogorov-Smirnov two sample test with $n, m > 10$,

$$\begin{cases} H_0 : F(x) = G(x) \ \forall x \in \Re. \\ H_1 : F(x) \neq G(x) \ for \ some \ x \in \Re. \end{cases}$$

If the null hypothesis is true, the population distributions are identical and we have two samples from the same population.

This test is based on the differences between the empirical distribution functions of the two samples. Under H_0, these deviations should be small and we can use them to do the test with the statistic $D_{n,m} = \max_{-\infty<x<\infty} |F_n(x) - G_m(x)|$, where $F_n(x)$ and $G_m(x)$ are the respective proportions of the two samples which are less than or equal to x. Given the significance level α, the critical region is $D_{n,m} > D_{n,m,\alpha}$.

It's known that $\lim_{n \longrightarrow \infty} \phi_n(z) = \sum_{k=-\infty}^{\infty} (-1)^k e^{-2k^2 z^2} = 1 - k(z)$ where $\phi_n(z) = p(D_n \leq \frac{z}{\sqrt{n}})$.

In the table of the asymptotic distribution of Kolmogorov-Smirnov statistic D_n, the probabilities $\lim_{n \longrightarrow \infty} p(D_n > \frac{z}{\sqrt{n}}) = k(z)$ are calculated. Also the asymptotic distribution of $\sqrt{\frac{nm}{n+m}} D_{n,m}$ is exactly the same as the asymptotic distribution of $\sqrt{n} D_n$.

Because the limit distribution is known, it's only necessary to calculate the sum of a serie using Sum[], generate the table with Table[] and put the style specifications *edit> preferences* of GridFrame with value true, RowLines with value {0.5,false} and ColumnLines with value true to obtain the output format. See table T1.

The code is,

```
<< "Statistics`"
<< "Graphics`"
<< "Statistics`DataManipulation`"
kol[k_,z_] :=(-1)^k*E^(-2*(k*z)^2)
Suma1[z_]:=1-Evaluate[Sum[kol[k,z],{k,-∞,∞}]]
T1=N[Table[{z,Suma1[z],z+0.44,Suma1[z+0.44],z+0.88,
Suma1[z+0.88],z+1.32,Suma1[z+1.32],z+1.76,Suma1[z+1.76]}, {z,0.32,0.75,0.01}],6];
T2=Prepend[T1,{"z(KS)","k(z)","z(KS)","k(z)","z(KS)","k(z)"
,"z(KS)","k(z)","z(KS)","k(z)"}];
StyleForm[TableForm[T2],FontSize -> 12]
```

2.2 Table of the exact distribution of statistic D_n

Given the significance level α, the critical region is $D_n > D_{n,\alpha}$. To obtain $d = D_{n,\alpha}$ that satisfies $P(D_n > D_{n,\alpha}) = \alpha$ we solve

$$P(D_n^+ > d) = (1-d)^n + d \sum_{j=1}^{[n(1-d)]} \binom{n}{j} (1 - d - \frac{j}{n})^{n-j} (d + \frac{j}{n})^{j-1} = \frac{\alpha}{2}$$

where $D_n^+ = max_x[F_n(x) - F_0(x)]$.

The values of the probabilities α are 0.2, 0.1, 0.05, 0.02 and 0.01. One can change these values to other values in the program. The previous result due to Birnbaum y Tingey (1951) is used to make these calculations. This formulation is computationally efficient for $n \leq 40$.

Five equations have been solved using FindRoot[] and a table has been built.

The style specifications are *edit> preferences*, GridFrame with true, RowLines {0.5,false} and ColumnLines {0.5,false} to obtain the table. See table T2.

The code is,

```
<< Statistics`
Do[
n1 = m;
f2[v_] = (1 - v)^n1 + v * ( Sum_{j=1}^{IntegerPart[n1*(1-v)]} (Binomial[n1, j] * (1 - v - j / n1)^(n1-j) (v + j / n1)^(j -1)) );
(*calcula el doble de lo pedido para Dn con D+*);
A[m] = Join[
{FindRoot[f2[v] == 0.1, {v, {0.1, 0.5}}]},
{FindRoot[f2[v] == 0.05, {v, {0.1, 0.5}}]},
{FindRoot[f2[v] == 0.025, {v, {0.1, 0.5}}]},
{FindRoot[f2[v] == 0.01, {v, {0.1, 0.5}}]},
{FindRoot[f2[v] == 0.005, {v, {0.1, 0.5}}]}];
T[m] = v /. A[m];
T2[m] = T[m];
T1[m] = Prepend[T2[m], n1];
Lin = {"n", "0.2", "0.1", "0.05", "0.02", "0.01"}, {m, 1, 40}];
  TT = Prepend[Table[T1[m], {m, 1, 40}], Lin];
TableForm[TT]
```

2.3 Table of the distribution of statistic $D_{m,n}$

In this case, the probabilities $P(mnD_{m,n} \geq d) = \alpha$ are calculated for $n, m = 2, .., 10$ and for $n = m$ from 10 to 15. This can be written $P(D_{m,n} \geq \frac{d}{mn}) = \alpha$, where $D_{m,n} = \max\limits_{-\infty<x<\infty} |F_m(x) - G_n(x)|$.

$$P(D_{m,n} \geq d) = 1 - \frac{a(m,n)}{\binom{m+n}{m}}$$

where a(m,n) satisfaces the recursion relation $a(m,n) = a(m-1,n) + a(m,n-1)$ with boundary conditions $a(0,n) = a(m,0) = 1$.

Do[], If[], Length[] are used. Furthermore, I use the symbolic lenguage of Mathematica to define functions. The specifications are *edit> preferences* GridFrame true, RowLines {0.5,false} and ColumnLines {0.5,false} to obtain the output format. See table T3.

The code is,

```
<< "Statistics`"
<< "DiscreteMath`"
pp1 = 1; m1 = 10; n1 = 10;
Do[Do[Do[
  m1 = p2;
  n1 = p1;
dis1 = p1 * d;
Do[
Inf[n, dis1] = Max[0, IntegerPart[(m1 * (n - dis1) / n1)] + 1];
If[m1 * (n - dis1) / n1 < 0, Inf[n, dis1] = 0];
If[n == dis1, Inf[n, dis1] = 1];
If[Inf[n, dis1] == (m1 * (n - dis1) / n1), Inf[n] = (m1 * (n - dis1) / n1) + 1];
Sup[n, dis1] = Min[m1, IntegerPart[(m1 * (n + dis1) / n1)]];
If[Sup[n, dis1] == (m1 * (n + dis1) / n1), Sup[n, dis1] = (m1 * (n + dis1) / n1) - 1];
(*Print[{Inf[n,dis1],Sup[n,dis1]}]*),
{n, 0, n1}];
Do[
Inf1[m, dis1] = Max[0, IntegerPart[(n1 * m / m1 - dis1)] + 1];
If[n1 * m / m1 - dis1 < 0, Inf1[m, dis1] = 0];
If[n1 * m / m1 == dis1, Inf1[m, dis1] = 1];
If[Inf1[m, dis1] == n1 * m / m1 - dis1, Inf1[m, dis1] = (n1 * m / m1 - dis1) + 1];
Sup1[m, dis1] = Min[n1, IntegerPart[(n1 * m / m1 + dis1)]];
If[Sup1[m, dis1] == (n1 * m / m1 + dis1), Sup1[m, dis1] = (n1 * m / m1 + dis1) - 1];
(*Print[{Inf1[m,dis1],Sup1[m,dis1]}]*),
{m, 0, m1}];
Do[a[m, 0, dis1] = 0, {m, 1, m1}];
Do[a[m, 0, dis1] = 1, {m, Inf[0, dis1], Sup[0, dis1]}];
Do[a[0, n, dis1] = 0, {n, 1, n1}];
Do[a[0, n, dis1] = 1, {n, Inf1[0, dis1], Sup1[0, dis1]}];
a[0, 0, dis1] = 0;
Do[Do[a[m, n, dis1] = 0, {m, 1, m1}], {n, 1, n1}];
Do[Do[a[m, n, dis1] = a[m - 1, n, dis1] + a[m, n - 1, dis1], {m, Max[Inf[n, dis1], 1], Sup[n, dis1]}], {n, 1, n1}];
Do[
Do[F[m, n, dis1] = a[m, n, dis1] / Binomial[m + n, m];
p[m, n, dis1] = 1 - F[m, n, dis1], {m, 0, m1}];
T1[n, dis1] = Table[a[m, n, dis1], {m, 0, m1}], {n, 0, n1}];
T2 = Table[T1[n, dis1], {n, 0, n1}],
{d, 0, 1, 1 / (m1 * p1)}];
T6[p1] = Prepend[Table[d * m1 * p1, {d, 0, 1, 1 / (p1)}], "d/n"];
(*Print[T6[p1]]*);
L6[p1] = Length[T6[p1]];
T4[p2, p1] = Table[N[p[p2, p1, d * p1], 4], {d, 0, 1, 1 / (p1)}];
L4[p1] = Length[T4[p2, p1]] + 1;
(*Print[T4[p2,p1]]*);
 T4[p2, n1] = Table[N[p[p2, n1, d * n1], 4], {d, 0, 1, 1 / (n1)}];
L44[p1] = Length[T4[p2, p1]] + 1,
 {p1, 1, n1}];
Do[L[p1] = L4[n1] - L44[p1], {p1, 1, n1}];
TT[p1_] := Table[" ", {k1, 1, L[p1]}];
T8 = Prepend[Table[Prepend[Join[T4[p2, p1], TT[p1]], p1], {p1, p2, n1}], T6[n1]];
 Print["m=", p2];
Print[TableForm[Transpose[T8]]]
, {p2, 2, m1}]
```

2.4 Table of the distribution of the runs R

It is known that the probability distribution of the number of runs R is,

$$p(R=r)=\begin{cases} \frac{2.\binom{n_1-1}{\frac{r}{2}-1}\cdot\binom{n_2-1}{\frac{r}{2}-1}}{\binom{n_1+n_2}{n_1}} & \text{if r is even .} \\ \frac{\binom{n_1-1}{\frac{r+1}{2}-1}\cdot\binom{n_2-1}{\frac{r-1}{2}-1}+\binom{n_1-1}{\frac{r-1}{2}-1}\cdot\binom{n_2-1}{\frac{r+1}{2}-1}}{\binom{n_1+n_2}{n_1}} & \text{if r is odd.} \end{cases}$$

This distribution is used in the test of randomness. It is one of the best known and easiest to apply. The data can be dichotomous or it can be classified into a dichotomous sequence. In this test based on the total number of runs, both too few and too many runs suggest lack of randomness. Given the level α, the critical region is $R \leq r_{\frac{\alpha}{2}}$ and $R \geq r'_{\frac{\alpha}{2}}$ that $p(R \leq r_{\frac{\alpha}{2}}) \leq \frac{\alpha}{2}$ and $p(R \geq r'_{\frac{\alpha}{2}}) \leq \frac{\alpha}{2}$.

This distribution can also be used to compare distributions in the Wald-Wolfowitz runs test. In this case R is the number of runs in the combined sample. Given the significance level α usually this test has the critical region $R \leq r_\alpha$, where $p(R \leq r_\alpha) \leq \alpha$

The table shows the values $P(R \geq r) = \alpha$ where r =2,3,...,n_1+n_2 for $n_1, n_2 \leq 10$. The probability distribution is defined in two parts for r even in $p1[n_1, n_2, rpar]$ and for r odd in $p2[n_1, n_2, rimpar]$, some sums are evaluated to obtain $p(R < r) = p(R \leq r-1)$ with Sum[] and the program saves in tables the probabilities of the complementary sucess $p(R \geq r)$.

The specifications are *edit> preferences* GridFrame true, RowLines {0.5,0.5,false} and ColumnLines {false,0.5,false} to obtain the output format. See table T4.

The code is,

```
<< "Statistics`"
<< "Graphics`"
<< "Statistics`DataManipulation`"
P1[n1_, n2_, rpar_] := (2*Binomial[n1 - 1, rpar / 2 - 1] *
  Binomial[n2 - 1, rpar / 2 - 1] ) / Binomial[n1 + n2, n1];
P2[n1_, n2_, rimpar_] := (Binomial[n1 - 1, (rimpar - 1) / 2] *
  Binomial[n2 - 1, (rimpar - 3) / 2] +
    Binomial[n1 - 1, (rimpar - 3) / 2] * Binomial[n2 - 1, (rimpar - 1) / 2] )
   / Binomial[n1 + n2, n1];
Sumas1[nr_] := Table[P1[n1, n2, nr], {n1, 2, 10}, {n2, n1, 10}];
Sumas2[nr_] := Table[P2[n1, n2, nr], {n1, 2, 10}, {n2, n1, 10}];
C1 = Flatten[Table[{n1, n2}, {n1, 2, 10}, {n2, n1, 10}], 1];
C11 = Prepend[Column[C1, 2], "n2"];
C12 = Prepend[Column[C1, 1], "n1"];
Sumatot[nrachas_] := Prepend[N[Flatten[Sum[Sumas1[nr], {nr, 2, nrachas, 2}] +
     Sum[Sumas2[nr], {nr, 3, nrachas, 2}]], 4], -nrachas];
T5 = Table[1 - Sumatot[k], {k, 2, 9}];
T4 = Prepend[Prepend[T5, C11], C12];
T1 = Transpose[T4];
T2 = Prepend[T1, {" ", "  ", " ", " ", " ", " ", "Rachas ", " ", " ", " "}];
StyleForm[TableForm[T2], FontSize -> 12]
T51 = Table[1 - Sumatot[k], {k, 10, 19}];
T41 = Prepend[Prepend[T51, C11], C12];
T11 = Transpose[T41];
T21 = Prepend[T11, {" ", "  ", " ", " ", " ", " ", "Rachas ", " ", " ", " "}];
StyleForm[TableForm[T21], FontSize -> 12]
```

2.5 Table of the distribution of Wilcoxon signed-rank statistic W^+

The Wilcoxon signed-rank test is a test of location,

$$\begin{cases} H_0 : M_e = m. \\ H_1 : M_e \neq m. \end{cases}$$

where M_e is the unknown median of the distribution.

The Wilcoxon signed-rank statistic W^+ gives the sum of the ranks of the positive differences where the differences are defined as $D_i = X_i - m$. If m is the true median of the symmetrical population, the expectation of the sum of the ranks of the positive differences W^+ equals the expectation of the sum of the ranks of the negative differences W^-. Given the significance level α, the critical region is $W^+ \leq k1$ and $W^+ \geq k2$ where $p(W^+ \leq k1) \leq \frac{\alpha}{2}$ and $p(W^+ \geq k2) \leq \frac{\alpha}{2}$.

The probabilities $p(W^+ \leq k1) = p(W^+ \geq k2)$ are calculated where $p(W^+ = k) = \frac{u(k)}{2^n}$, where u(k) is the number of ways to assign plus and minus signs to the first n integers such that the sum of the positive integers equals k, with sample size $n \leq 16$.

To calculate u(k) the commands GrayCode and Frequencies are used. And finally, to obtain $p(W^+ \leq k1)$ I use the command Sum[] to sum probabilities. See table T5.

The specifications are *edit> preferences* GridFrame false, RowLines false and ColumnLines false to obtain the output format.

The code is,

```
<< Statistics`
<< DiscreteMath`
Share[]
(*n=5*)
Do[
 Partes[n] = GrayCode[Range[n]];
L[n] = Length[Partes[n]];
T1[n] = Table[Apply[Plus, Extract[Partes[n], i]], {i, 1, L[n]}];
F1[n] = Frequencies[T1[n]];
rango[n] = Column[F1[n], 2];
freq[n] = Column[F1[n], 1];
prob[n] = N[freq[n] / 2^n, 4];
rango1[n] = rango[n];
F[n_, k_] := Sum[prob[n][[m]], {m, 1, k}];
Lim[n] = n * (n + 1) / 2;
c[n] = IntegerPart[n * (n + 1) / 4];
TC1[n] = Table[F[n, nc], {nc, 1, c[n] + 1}];
S1[n] = Table[c - 1, {c, 1, c[n] + 1}];
S2[n] = Table[Lim[n] - c + 1, {c, 1, c[n] + 1}];
TOT = Table[{n, S1[n], TC1[n], S2[n]}, {n, n, n}];
TOT2 = Prepend[TOT, {"__", "__ ", "________ ", "__ "}];
TOT1 = Prepend[Prepend[TOT2, {" ", " ", "p(W⁺≥k2)", "  "}],
   {"n", "k1 ", "p(W⁺≤k1) ", "k2"}];
TOT3 = Append[
   Append[TOT1, {" ", " ", " ", " "}], {" ", " ", " ", " "}];
Print[TableForm[TOT3]], {n, 2, 16, 1}]
```

2.6 Table of distribution of de Mann-Whitney statistic U_x

The Mann-Whitney U test is used to compare distributions,

$$\begin{cases} H_0 : F(x) = G(x) \ \forall x \in \Re. \\ H_1 : F(x) \neq G(x) \ for\ some\ x \in \Re. \end{cases}$$

The Mann-Whitney U test is based on the idea that the particular pattern exhibited when the samples of the random variables X and Y are arranged together in increasing order of magnitude. It provides information about the relationship between their populations. Mann-Whitney statistic U_x is defined as the number of times an X precedes a Y in the combined ordered sample,

$$U_x = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} Z_{ij} \qquad where \qquad Z_{ij} = \begin{cases} 1 & \text{si } X_i < Y_j \\ 0 & \text{si } X_i > Y_j \\ \frac{1}{2} & \text{si } X_i = Y_j \end{cases}$$

Given α, the critical region is $U_x \leq u_1$ and $U_x \geq u_2$ where $p(U_x \leq u_1) \leq \frac{\alpha}{2}$ and $p(U_x \geq u_2) \leq \frac{\alpha}{2}$.

The probabilities $p(U_x \leq k)$ are calculated for $n_1 \leq n_2$ and n_1=$2, 3, ..., 10$.

Firstly, the probabilities $p(U_x = k)$ are calculated using the recursion relation for m from 1 to n_1 and n from 1 to n_2,

$$p_{m,n}(k) = \frac{n}{n+m} p_{m,n-1}(k) + \frac{m}{n+m} p_{m-1,n}(k-n)$$

where k takes values from 0 to $n_1 n_2$ and the boundary conditions are defined as follow,

$p_{m,n}(k) = 0$ if $k < 0$

$p_{m,0}(0) = p_{0,n}(0) = 1$

$p_{m,n}(k) = 0$ if $k < 0$

$p_{m,0}(k) = p_{0,n}(k) = 0$ if $k \neq 0$.

The probabilities $p(U_x \leq k)$ are calculated with Sum[]. The specifications are *edit> preferences* GridFrame true, RowLines {0.5,0.5,false} and ColumnLines {0.5,false} to obtain the output format. See table T6.

The code is,

```
<< "Statistics`"
<< "DiscreteMath`"
pp1 = 1; nn1 = 2; nn2 = 10;
Do[p[0, n, u] = 0, {n, 1, nn2}, {u, -nn1*nn2, -1, 1}];
Do[
Do[
(*nm1=n1*m1/2;*)
nm1 = n1*m1/2;
Do[p[m, n, u] = 0, {m, 1, m1}, {n, 1, n1}, {u, -nm1, -1, 1}];
Do[p[m, 0, u] = 0, {m, 1, m1}, {u, 1, nm1}];
Do[p[m, 0, u] = 0, {m, 1, m1}, {u, -nm1, -1, 1}];
Do[p[0, n, u] = 0, {n, 1, n1}, {u, 1, nm1}];
Do[p[0, n, u] = 0, {n, 1, n1}, {u, -nm1, -1, 1}];
Do[p[m, 0, 0] = 1, {m, 1, m1}];
Do[p[0, n, 0] = 1, {n, 1, n1}];
Do[p[m, n, u] =
    (n/(n+m))*p[m, n-1, u] + (m/(n+m))*p[m-1, n, u-n], {m, 1, m1}, {n, 1, n1}, {u, 0, nm1}];
p2[m1, n1] = N[Table[p[m1, n1, u], {u, 0, IntegerPart[nm1]}], 4];
F1[k_] := Sum[p2[m1, n1][[u1]], {u1, 1, k+1}];
 T4 = Prepend[Join[Table[k, {k, 0, IntegerPart[nm1]}],
     Table[" ", {k, 0, IntegerPart[n1*n1/2] - IntegerPart[nm1]}]], "k/n1"];
  T1[m1, n1] = Join[Table[F1[k], {k, 0, IntegerPart[nm1]}],
    Table[" ", {k, 0, IntegerPart[n1*n1/2] - IntegerPart[nm1]}]];
Do[p[m, n, u] = 0, {m, 1, m1}, {n, 1, n1}, {u, -nm1, nm1, 1}],
 {m1, 1, n1}];
T2[n1] = Transpose[Prepend[Table[Prepend[T1[m1, n1], m1],  {m1, 1, n1}], T4]];
Print[TableForm[Prepend[T2[n1], {"n2", n1}]]],
 {n1, nn1, nn2}]
```

3 Computational results. Tables.

3.1 TABLE T1. asymptotic Table for D_n.

$$\lim_{n\longrightarrow\infty} p(\sqrt{n}D_n > z(KS)) = \lim_{n\longrightarrow\infty} p(D_n > \frac{z(KS)}{\sqrt{n}}) = k(z)$$

z(KS)	k(z)	z(KS)	k(z)	z(KS)	k(z)	z(KS)	k(z)	z(KS)	k(z)
0.32	0.999954	0.76	0.61036	1.2	0.11225	1.64	0.00922302	2.08	0.000349274
0.33	0.999909	0.77	0.593628	1.21	0.10697	1.65	0.00863568	2.09	0.000321326
0.34	0.999829	0.78	0.576998	1.22	0.101898	1.66	0.00808251	2.1	0.000295497
0.35	0.999697	0.79	0.560495	1.23	0.0970269	1.67	0.00756175	2.11	0.000271635
0.36	0.999489	0.8	0.544142	1.24	0.0923517	1.68	0.00707171	2.12	0.0002496
0.37	0.999174	0.81	0.527961	1.25	0.0878664	1.69	0.00661079	2.13	0.00022926
0.38	0.998715	0.82	0.511972	1.26	0.0835654	1.7	0.00617743	2.14	0.000210494
0.39	0.998071	0.83	0.496191	1.27	0.079443	1.71	0.00577018	2.15	0.000193187
0.4	0.997192	0.84	0.480635	1.28	0.0754937	1.72	0.00538761	2.16	0.000177232
0.41	0.996028	0.85	0.465319	1.29	0.0717119	1.73	0.0050284	2.17	0.000162529
0.42	0.994524	0.86	0.450255	1.3	0.0680922	1.74	0.00469127	2.18	0.000148987
0.43	0.992623	0.87	0.435455	1.31	0.0646294	1.75	0.00437498	2.19	0.000136518
0.44	0.99027	0.88	0.420929	1.32	0.061318	1.76	0.00407839	2.2	0.000125043
0.45	0.987411	0.89	0.406685	1.33	0.058153	1.77	0.00380039	2.21	0.000114487
0.46	0.983995	0.9	0.392731	1.34	0.0551293	1.78	0.00353991	2.22	0.000104779
0.47	0.979978	0.91	0.379072	1.35	0.0522419	1.79	0.00329598	2.23	0.000095857
0.48	0.975318	0.92	0.365715	1.36	0.0494859	1.8	0.00306762	2.24	0.0000876593
0.49	0.969983	0.93	0.352663	1.37	0.0468565	1.81	0.00285395	2.25	0.0000801306
0.5	0.963945	0.94	0.339919	1.38	0.0443491	1.82	0.00265409	2.26	0.0000732192
0.51	0.957186	0.95	0.327485	1.39	0.041959	1.83	0.00246725	2.27	0.0000668772
0.52	0.949694	0.96	0.315364	1.4	0.0396819	1.84	0.00229264	2.28	0.00006106
0.53	0.941466	0.97	0.303555	1.41	0.0375133	1.85	0.00212953	2.29	0.0000557266
0.54	0.932503	0.98	0.292059	1.42	0.0354491	1.86	0.00197724	2.3	0.0000508387
0.55	0.922817	0.99	0.280874	1.43	0.033485	1.87	0.00183511	2.31	0.000046361
0.56	0.912423	1.	0.27	1.44	0.0316171	1.88	0.00170251	2.32	0.0000422607
0.57	0.901344	1.01	0.259434	1.45	0.0298415	1.89	0.00157886	2.33	0.0000385077
0.58	0.889606	1.02	0.249175	1.46	0.0281543	1.9	0.0014636	2.34	0.0000350739
0.59	0.87724	1.03	0.239219	1.47	0.0265519	1.91	0.00135622	2.35	0.0000319336
0.6	0.864283	1.04	0.229564	1.48	0.0250306	1.92	0.00125621	2.36	0.0000290628
0.61	0.850771	1.05	0.220206	1.49	0.0235871	1.93	0.00116312	2.37	0.0000264395
0.62	0.836745	1.06	0.21114	1.5	0.022218	1.94	0.00107649	2.38	0.0000240433
0.63	0.822248	1.07	0.202363	1.51	0.0209199	1.95	0.000995911	2.39	0.0000218556
0.64	0.807323	1.08	0.19387	1.52	0.0196898	1.96	0.000920998	2.4	0.000019859
0.65	0.792013	1.09	0.185657	1.53	0.0185246	1.97	0.000851379	2.41	0.0000180376
0.66	0.776363	1.1	0.177718	1.54	0.0174214	1.98	0.000786708	2.42	0.0000163767
0.67	0.760418	1.11	0.170049	1.55	0.0163774	1.99	0.000726659	2.43	0.0000148627
0.68	0.74422	1.12	0.162644	1.56	0.0153898	2.	0.000670925	2.44	0.0000134834
0.69	0.727812	1.13	0.155498	1.57	0.0144559	2.01	0.000619218	2.45	0.0000122271
0.7	0.711235	1.14	0.148605	1.58	0.0135733	2.02	0.000571268	2.46	0.0000110835
0.71	0.69453	1.15	0.14196	1.59	0.0127394	2.03	0.000526819	2.47	0.0000100428
0.72	0.677735	1.16	0.135557	1.6	0.011952	2.04	0.000485635	2.48	9.09621×10^{-6}
0.73	0.660886	1.17	0.12939	1.61	0.0112088	2.05	0.000447492	2.49	8.23553×10^{-6}
0.74	0.644019	1.18	0.123454	1.62	0.0105076	2.06	0.000412179	2.5	7.45331×10^{-6}
0.75	0.627167	1.19	0.117742	1.63	0.00984636	2.07	0.000379501	2.51	6.74268×10^{-6}

3.2 TABLE T2. Table for D_n, $n \leq 40$.

$$p(D_n > d) = \alpha$$

n	0.2	0.1	0.05	0.02	0.01
1	0.9	0.95	0.975	0.99	0.995
2	0.6838	0.7764	0.8419	0.9	0.9293
3	0.5648	0.636	0.7076	0.7846	0.829
4	0.4927	0.5652	0.6239	0.6889	0.7342
5	0.447	0.5094	0.5633	0.6272	0.6685
6	0.4104	0.468	0.5193	0.5774	0.6166
7	0.3815	0.4361	0.4834	0.5384	0.5758
8	0.3583	0.4096	0.4543	0.5065	0.5418
9	0.3391	0.3875	0.43	0.4796	0.5133
10	0.3226	0.3687	0.4092	0.4566	0.4889
11	0.3083	0.3524	0.3912	0.4367	0.4677
12	0.2958	0.3382	0.3754	0.4192	0.449
13	0.2847	0.3255	0.3614	0.4036	0.4325
14	0.2748	0.3142	0.3489	0.3897	0.4176
15	0.2659	0.304	0.3376	0.3771	0.4042
16	0.2578	0.2947	0.3273	0.3657	0.392
17	0.2504	0.2863	0.318	0.3553	0.3809
18	0.2436	0.2785	0.3094	0.3457	0.3706
19	0.2373	0.2714	0.3014	0.3369	0.3612
20	0.2316	0.2647	0.2941	0.3287	0.3524
21	0.2262	0.2586	0.2872	0.321	0.3443
22	0.2212	0.2528	0.2809	0.3139	0.3367
23	0.2165	0.2475	0.2749	0.3073	0.3295
24	0.212	0.2424	0.2693	0.301	0.3229
25	0.2079	0.2377	0.264	0.2952	0.3166
26	0.204	0.2332	0.2591	0.2896	0.3106
27	0.2003	0.229	0.2544	0.2844	0.305
28	0.1968	0.225	0.2499	0.2794	0.2997
29	0.1935	0.2212	0.2457	0.2747	0.2947
30	0.1903	0.2176	0.2417	0.2702	0.2899
31	0.1873	0.2141	0.2379	0.266	0.2853
32	0.1844	0.2108	0.2342	0.2619	0.2809
33	0.1817	0.2077	0.2308	0.258	0.2768
34	0.1791	0.2047	0.2274	0.2543	0.2728
35	0.1766	0.2018	0.2242	0.2507	0.269
36	0.1742	0.1991	0.2212	0.2473	0.2653
37	0.1719	0.1965	0.2183	0.244	0.2618
38	0.1697	0.1939	0.2154	0.2409	0.2584
39	0.1675	0.1915	0.2127	0.2379	0.2552
40	0.1655	0.1891	0.2101	0.2349	0.2521

If $n > 40$, use table T1 with $\frac{z(KS)}{\sqrt{n}}$.

3.3 TABLE T3. Table for $D_{m,n}$, n,m$\leq$ 10 and n=m 10 to 15.

$$P(mnD_{m,n} \geq d) = \alpha \Longrightarrow P(D_{m,n} \geq \frac{d}{mn}) = \alpha$$

m=2

d/n	2	3	4	5	6	7	8	9	10
0	1.	1.	1.	1.	1.	1.	1.	1.	1.
2	1.	1.	1.	1.	1.	1.	1.	1.	1.
4	0.3333	0.6	0.9333	0.9524	1.	1.	1.	1.	1.
6		0.2	0.4	0.5714	0.8571	0.8889	0.9778	0.9818	1.
8			0.1333	0.2857	0.4286	0.5556	0.8	0.8364	0.9394
10				0.09524	0.2143	0.3333	0.4444	0.5455	0.7576
12					0.07143	0.1667	0.2667	0.3636	0.4545
14						0.05556	0.1333	0.2182	0.303
16							0.04444	0.1091	0.1818
18								0.03636	0.09091
20									0.0303

m=3

d/n	3	4	5	6	7	8	9	10
0	1.	1.	1.	1.	1.	1.	1.	1.
3	1.	1.	1.	1.	1.	1.	1.	1.
6	0.6	0.6571	0.8571	0.9881	0.9833	0.9939	1.	1.
9	0.1	0.2286	0.4643	0.6786	0.7	0.8364	0.9636	0.958
12		0.05714	0.1429	0.3333	0.4	0.5636	0.7091	0.7203
15			0.03571	0.09524	0.1667	0.303	0.4545	0.493
18				0.02381	0.06667	0.1212	0.2364	0.2867
21					0.01667	0.04848	0.09091	0.1399
24						0.01212	0.03636	0.06993
27							0.009091	0.02797
30								0.006993

m=4

d/n	4	5	6	7	8	9	10
0	1.	1.	1.	1.	1.	1.	1.
4	1.	1.	1.	1.	1.	1.	1.
8	0.7714	0.746	0.9238	0.9515	0.998	0.9944	0.999
12	0.2286	0.2857	0.5524	0.6606	0.8364	0.8112	0.9191
16	0.02857	0.07937	0.181	0.3091	0.5131	0.5175	0.6873
20		0.01587	0.04762	0.1212	0.2222	0.2517	0.4096
24			0.009524	0.0303	0.08485	0.1147	0.1878
28				0.006061	0.0202	0.04196	0.08392
32					0.00404	0.01399	0.02997
36						0.002797	0.00999
40							0.001998

Continuation

m=5

d/n	5	6	7	8	9	10
0	1.	1.	1.	1.	1.	1.
5	1.	1.	1.	1.	1.	1.
10	0.873	0.8182	0.9091	0.9627	0.984	0.9997
15	0.3571	0.3571	0.5455	0.6838	0.7912	0.9191
20	0.07937	0.1082	0.2374	0.3162	0.4605	0.6547
25	0.007937	0.02597	0.06566	0.1259	0.2258	0.3506
30		0.004329	0.01515	0.04196	0.08591	0.1658
35			0.002525	0.009324	0.02797	0.06061
40				0.001554	0.005994	0.01931
45					0.000999	0.003996
50						0.000666

m=6

d/n	6	7	8	9	10
0	1.	1.	1.	1.	1.
6	1.	1.	1.	1.	1.
12	0.9307	0.8718	0.952	0.9872	0.992
18	0.474	0.4336	0.6374	0.7794	0.8354
24	0.1429	0.1469	0.3017	0.4084	0.5055
30	0.02597	0.03846	0.09257	0.1758	0.2488
36	0.002165	0.008159	0.02264	0.06114	0.09216
42		0.001166	0.004662	0.01399	0.03147
48			0.000666	0.002797	0.008991
54				0.0003996	0.001748
60					0.0002498

m=7

d/n	7	8	9	10
0	1.	1.	1.	1.
7	1.	1.	1.	1.
14	0.9627	0.9105	0.9552	0.9778
21	0.5752	0.5077	0.6476	0.7533
28	0.2121	0.1925	0.3238	0.4334
35	0.05303	0.05594	0.1267	0.1894
42	0.008159	0.01305	0.03409	0.07158
49	0.0005828	0.002486	0.007517	0.0217
56		0.0003108	0.001399	0.004525
63			0.0001748	0.0008227
70				0.0001028

m=8

d/n	8	9	10
0	1.	1.	1.
8	1.	1.	1.
16	0.9801	0.938	0.9766
24	0.6601	0.5761	0.7227
32	0.2827	0.2425	0.3917
40	0.08702	0.07857	0.166
48	0.01865	0.02024	0.04987
56	0.002486	0.004278	0.01202
64	0.0001554	0.0007404	0.002422
72		0.00008227	0.0004114
80			0.00004571

Continuation

m=9

d/n	9	10
0	1.	1.
9	1.	1.
18	0.9895	0.9573
27	0.7301	0.6375
36	0.3517	0.2942
45	0.1259	0.1056
54	0.03357	0.03027
63	0.006294	0.007036
72	0.0007404	0.001364
81	0.00004114	0.0002165
90		0.00002165

m=10

d/n	10
0	1.
10	1.
20	0.9945
30	0.7869
40	0.4175
50	0.1678
60	0.05245
70	0.01234
80	0.002057
90	0.0002165
100	0.00001083

If m=n from 10 to 15,

m=10

d/n	10
0	1.
10	1.
20	0.9945
30	0.7869
40	0.4175
50	0.1678
60	0.05245
70	0.01234
80	0.002057
90	0.0002165
100	0.00001083

m=11

d/n	11
0	1.
11	1.
22	0.9971
33	0.8326
44	0.4792
55	0.2115
66	0.07466
77	0.02074
88	0.004366
99	0.0006549
110	0.00006237
121	2.835×10^{-6}

m=12

d/n	12
0	1.
12	1.
24	0.9985
36	0.869
48	0.5361
60	0.2558
72	0.09955
84	0.03144
96	0.007859
108	0.001497
120	0.0002041
132	0.00001775
144	7.396×10^{-7}

m=13

d/n	13
0	1.
13	1.
26	0.9992
39	0.8978
52	0.5882
65	0.2999
78	0.1265
91	0.04427
104	0.01265
117	0.002875
130	0.0005
143	0.0000625
156	$5.\times10^{-6}$
169	1.923×10^{-7}

m=14

d/n	14
0	1.
14	1.
28	0.9996
42	0.9205
56	0.6355
70	0.3433
84	0.1549
98	0.05903
112	0.01878
126	0.0049
140	0.001021
154	0.0001633
168	0.00001885
182	1.396×10^{-6}
196	4.985×10^{-8}

m=15

d/n	15
0	1.
15	1.
30	0.9998
45	0.9383
60	0.6781
75	0.3855
90	0.1844
105	0.07546
120	0.02625
135	0.007656
150	0.001837
165	0.0003533
180	0.00005235
195	5.609×10^{-6}
210	3.868×10^{-7}
225	1.289×10^{-8}

For $n, m > 10$, table T1 with $z(KS)\sqrt{\frac{n+m}{nm}}$.

3.4 TABLE T4. Table for R, $n_1, n_2 \leq 10$.

$$P(R \geq r) = \alpha \; where \; r = \; 2, \; 3 \; , ..., \; n_1 + n_2$$

		Rachas							
n1	n2	3	4	5	6	7	8	9	10
2	2	0.6667	0.3333	0.	0.	0.	0.	0.	0.
2	3	0.8	0.5	0.1	0.	0.	0.	0.	0.
2	4	0.8667	0.6	0.2	0.	0.	0.	0.	0.
2	5	0.9048	0.6667	0.2857	0.	0.	0.	0.	0.
2	6	0.9286	0.7143	0.3571	0.	0.	0.	0.	0.
2	7	0.9444	0.75	0.4167	0.	0.	0.	0.	0.
2	8	0.9556	0.7778	0.4667	0.	0.	0.	0.	0.
2	9	0.9636	0.8	0.5091	0.	0.	0.	0.	0.
2	10	0.9697	0.8182	0.5455	0.	0.	0.	0.	0.
3	3	0.9	0.7	0.3	0.1	0.	0.	0.	0.
3	4	0.9429	0.8	0.4571	0.2	0.02857	0.	0.	0.
3	5	0.9643	0.8571	0.5714	0.2857	0.07143	0.	0.	0.
3	6	0.9762	0.8929	0.6548	0.3571	0.119	0.	0.	0.
3	7	0.9833	0.9167	0.7167	0.4167	0.1667	0.	0.	0.
3	8	0.9879	0.9333	0.7636	0.4667	0.2121	0.	0.	0.
3	9	0.9909	0.9455	0.8	0.5091	0.2545	0.	0.	0.
3	10	0.993	0.9545	0.8287	0.5455	0.2937	0.	0.	0.
4	4	0.9714	0.8857	0.6286	0.3714	0.1143	0.02857	0.	0.
4	5	0.9841	0.9286	0.7381	0.5	0.2143	0.07143	0.007937	0.
4	6	0.9905	0.9524	0.8095	0.5952	0.3095	0.119	0.02381	0.
4	7	0.9939	0.9667	0.8576	0.6667	0.3939	0.1667	0.04545	0.
4	8	0.996	0.9758	0.8909	0.7212	0.4667	0.2121	0.07071	0.
4	9	0.9972	0.9818	0.9147	0.7636	0.5287	0.2545	0.0979	0.
4	10	0.998	0.986	0.9321	0.7972	0.5814	0.2937	0.1259	0.
5	5	0.9921	0.9603	0.8333	0.6429	0.3571	0.1667	0.03968	0.007937
5	6	0.9957	0.9762	0.8896	0.7381	0.4784	0.2619	0.08874	0.02381
5	7	0.9975	0.9848	0.9242	0.803	0.5758	0.3485	0.1465	0.04545
5	8	0.9984	0.9899	0.9464	0.8485	0.6527	0.4242	0.2067	0.07071
5	9	0.999	0.993	0.961	0.8811	0.7133	0.4895	0.2657	0.0979
5	10	0.9993	0.995	0.971	0.9051	0.7612	0.5455	0.3217	0.1259
6	6	0.9978	0.987	0.9329	0.8247	0.6082	0.3918	0.1753	0.0671
6	7	0.9988	0.9924	0.9575	0.8788	0.704	0.5	0.2669	0.1212
6	8	0.9993	0.9953	0.972	0.9138	0.7739	0.5874	0.3543	0.1795
6	9	0.9996	0.997	0.981	0.9371	0.8252	0.6573	0.4336	0.2378
6	10	0.9998	0.998	0.9868	0.953	0.8631	0.7133	0.5035	0.2937
7	7	0.9994	0.9959	0.9749	0.9225	0.7914	0.6166	0.3834	0.2086
7	8	0.9997	0.9977	0.9846	0.9487	0.8508	0.704	0.4864	0.296
7	9	0.9998	0.9986	0.9902	0.965	0.8916	0.7692	0.5734	0.3776
7	10	0.9999	0.9991	0.9936	0.9755	0.92	0.8182	0.6454	0.451
8	8	0.9998	0.9988	0.9911	0.9683	0.8998	0.7855	0.5952	0.4048
8	9	0.9999	0.9993	0.9947	0.9797	0.9313	0.8427	0.6814	0.5
8	10	1.	0.9996	0.9967	0.9866	0.9521	0.883	0.7486	0.5806
9	9	1.	0.9996	0.997	0.9878	0.9555	0.891	0.762	0.6008
9	10	1.	0.9998	0.9982	0.9924	0.9706	0.9233	0.8214	0.6814
10	10	1.	0.9999	0.999	0.9955	0.9815	0.9487	0.8724	0.7578

Continuation

		Rachas									
n1	n2	11	12	13	14	15	16	17	18	19	20
2	2	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	4	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	8	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	9	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
2	10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	3	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	4	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	8	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	9	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
3	10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	4	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	6	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	7	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	8	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	9	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
4	10	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	5	0.	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	6	0.002165	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	7	0.007576	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	8	0.01632	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	9	0.02797	0.	0.	0.	0.	0.	0.	0.	0.	0.
5	10	0.04196	0.	0.	0.	0.	0.	0.	0.	0.	0.
6	6	0.01299	0.002165	0.	0.	0.	0.	0.	0.	0.	0.
6	7	0.0338	0.007576	0.0005828	0.	0.	0.	0.	0.	0.	0.
6	8	0.06294	0.01632	0.002331	0.	0.	0.	0.	0.	0.	0.
6	9	0.0979	0.02797	0.005594	0.	0.	0.	0.	0.	0.	0.
6	10	0.1364	0.04196	0.01049	0.	0.	0.	0.	0.	0.	0.
7	7	0.07751	0.02506	0.004079	0.0005828	0.	0.	0.	0.	0.	0.
7	8	0.1329	0.05128	0.01212	0.002331	0.0001554	0.	0.	0.	0.	0.
7	9	0.1941	0.08392	0.02517	0.005594	0.0006993	0.	0.	0.	0.	0.
7	10	0.2567	0.1206	0.04288	0.01049	0.001851	0.	0.	0.	0.	0.
8	8	0.2145	0.1002	0.0317	0.008858	0.001243	0.0001554	0.	0.	0.	0.
8	9	0.2984	0.1573	0.06059	0.02028	0.004155	0.0006993	0.00004114	0.	0.	0.
8	10	0.3791	0.2178	0.09687	0.0364	0.00953	0.001851	0.0002057	0.	0.	0.
9	9	0.3992	0.238	0.109	0.04447	0.01222	0.003003	0.0003702	0.00004114	0.	0.
9	10	0.4905	0.3186	0.1658	0.07672	0.0258	0.00761	0.001375	0.0002057	0.00001083	0.
10	10	0.5859	0.4141	0.2422	0.1276	0.05126	0.01852	0.004492	0.0009851	0.0001083	0.00001083

3.5 TABLE T5. Table for W^+.

$$p(W^+ \le k1) = p(W^+ \ge k2)$$

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
2	0	0.25	3
	1	0.5	2

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
3	0	0.125	6
	1	0.25	5
	2	0.375	4
	3	0.625	3

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
4	0	0.0625	10
	1	0.125	9
	2	0.1875	8
	3	0.3125	7
	4	0.4375	6
	5	0.5625	5

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
5	0	0.03125	15
	1	0.0625	14
	2	0.09375	13
	3	0.1563	12
	4	0.2188	11
	5	0.3125	10
	6	0.4063	9
	7	0.5	8

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
6	0	0.01563	21
	1	0.03125	20
	2	0.04688	19
	3	0.07813	18
	4	0.1094	17
	5	0.1563	16
	6	0.2188	15
	7	0.2813	14
	8	0.3438	13
	9	0.4219	12
	10	0.5	11

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
7	0	0.007813	28
	1	0.01563	27
	2	0.02344	26
	3	0.03906	25
	4	0.05469	24
	5	0.07813	23
	6	0.1094	22
	7	0.1484	21
	8	0.1875	20
	9	0.2344	19
	10	0.2891	18
	11	0.3438	17
	12	0.4063	16
	13	0.4688	15
	14	0.5313	14

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
8	0	0.003906	36
	1	0.007813	35
	2	0.01172	34
	3	0.01953	33
	4	0.02734	32
	5	0.03906	31
	6	0.05469	30
	7	0.07422	29
	8	0.09766	28
	9	0.125	27
	10	0.1563	26
	11	0.1914	25
	12	0.2305	24
	13	0.2734	23
	14	0.3203	22
	15	0.3711	21
	16	0.4219	20
	17	0.4727	19
	18	0.5273	18

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
9	0	0.001953	45
	1	0.003906	44
	2	0.005859	43
	3	0.009766	42
	4	0.01367	41
	5	0.01953	40
	6	0.02734	39
	7	0.03711	38
	8	0.04883	37
	9	0.06445	36
	10	0.08203	35
	11	0.1016	34
	12	0.125	33
	13	0.1504	32
	14	0.1797	31
	15	0.2129	30
	16	0.248	29
	17	0.2852	28
	18	0.3262	27
	19	0.3672	26
	20	0.4102	25
	21	0.4551	24
	22	0.5	23

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
10	0	0.0009766	55
	1	0.001953	54
	2	0.00293	53
	3	0.004883	52
	4	0.006836	51
	5	0.009766	50
	6	0.01367	49
	7	0.01855	48
	8	0.02441	47
	9	0.03223	46
	10	0.04199	45
	11	0.05273	44
	12	0.06543	43
	13	0.08008	42
	14	0.09668	41
	15	0.1162	40
	16	0.1377	39
	17	0.1611	38
	18	0.1875	37
	19	0.2158	36
	20	0.2461	35
	21	0.2783	34
	22	0.3125	33
	23	0.3477	32
	24	0.3848	31
	25	0.4229	30
	26	0.4609	29
	27	0.5	28

Continuation

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.0004883	66
	1	0.0009766	65
	2	0.001465	64
	3	0.002441	63
	4	0.003418	62
	5	0.004883	61
	6	0.006836	60
	7	0.009277	59
	8	0.01221	58
	9	0.01611	57
	10	0.021	56
	11	0.02686	55
	12	0.03369	54
	13	0.0415	53
	14	0.05078	52
	15	0.06152	51
11	16	0.07373	50
	17	0.0874	49
	18	0.103	48
	19	0.1201	47
	20	0.1392	46
	21	0.1602	45
	22	0.1826	44
	23	0.2065	43
	24	0.2324	42
	25	0.2598	41
	26	0.2886	40
	27	0.3188	39
	28	0.3501	38
	29	0.3823	37
	30	0.4155	36
	31	0.4492	35
	32	0.4829	34
	33	0.5171	33

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.0001221	91
	1	0.0002441	90
	2	0.0003662	89
	3	0.0006104	88
	4	0.0008545	87
	5	0.001221	86
	6	0.001709	85
	7	0.002319	84
	8	0.003052	83
	9	0.004028	82
	10	0.005249	81
	11	0.006714	80
	12	0.008545	79
	13	0.01074	78
	14	0.01331	77
	15	0.01636	76
	16	0.0199	75
	17	0.02393	74
	18	0.02869	73
	19	0.03406	72
	20	0.04016	71
	21	0.04712	70
13	22	0.05493	69
	23	0.0636	68
	24	0.07324	67
	25	0.08386	66
	26	0.09546	65
	27	0.1082	64
	28	0.1219	63
	29	0.1367	62
	30	0.1527	61
	31	0.1698	60
	32	0.1879	59
	33	0.2072	58
	34	0.2274	57
	35	0.2487	56
	36	0.2709	55
	37	0.2939	54
	38	0.3177	53
	39	0.3424	52
	40	0.3677	51
	41	0.3934	50
	42	0.4197	49
	43	0.4463	48
	44	0.473	47
	45	0.5	46

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.0002441	78
	1	0.0004883	77
	2	0.0007324	76
	3	0.001221	75
	4	0.001709	74
	5	0.002441	73
	6	0.003418	72
	7	0.004639	71
	8	0.006104	70
	9	0.008057	69
	10	0.0105	68
	11	0.01343	67
	12	0.01709	66
	13	0.02124	65
	14	0.02612	64
	15	0.03198	63
	16	0.03857	62
	17	0.04614	61
	18	0.05493	60
12	19	0.0647	59
	20	0.07568	58
	21	0.08813	57
	22	0.1018	56
	23	0.1167	55
	24	0.1331	54
	25	0.1506	53
	26	0.1697	52
	27	0.1902	51
	28	0.2119	50
	29	0.2349	49
	30	0.2593	48
	31	0.2847	47
	32	0.311	46
	33	0.3386	45
	34	0.3667	44
	35	0.3955	43
	36	0.425	42
	37	0.4548	41
	38	0.4849	40
	39	0.5151	39

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.00006104	105
	1	0.0001221	104
	2	0.0001831	103
	3	0.0003052	102
	4	0.0004272	101
	5	0.0006104	100
	6	0.0008545	99
	7	0.00116	98
	8	0.001526	97
	9	0.002014	96
	10	0.002625	95
	11	0.003357	94
	12	0.004272	93
	13	0.005371	92
	14	0.006714	91
	15	0.008301	90
	16	0.01013	89
	17	0.01227	88
	18	0.01477	87
	19	0.01764	86
	20	0.02094	85
	21	0.02472	84
	22	0.02899	83
	23	0.03381	82
	24	0.03925	81
	25	0.04529	80
14	26	0.052	79
	27	0.05945	78
	28	0.06763	77
	29	0.07654	76
	30	0.0863	75
	31	0.09686	74
	32	0.1083	73
	33	0.1206	72
	34	0.1338	71
	35	0.1479	70
	36	0.1629	69
	37	0.1788	68
	38	0.1955	67
	39	0.2131	66
	40	0.2316	65
	41	0.2508	64
	42	0.2708	63
	43	0.2915	62
	44	0.3129	61
	45	0.3349	60
	46	0.3574	59
	47	0.3804	58
	48	0.4039	57
	49	0.4276	56
	50	0.4516	55
	51	0.4758	54
	52	0.5	53

Continuation

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.00003052	120
	1	0.00006104	119
	2	0.00009155	118
	3	0.0001526	117
	4	0.0002136	116
	5	0.0003052	115
	6	0.0004272	114
	7	0.0005798	113
	8	0.0007629	112
	9	0.001007	111
	10	0.001312	110
	11	0.001678	109
	12	0.002136	108
	13	0.002686	107
	14	0.003357	106
	15	0.004181	105
	16	0.005127	104
	17	0.006226	103
	18	0.007538	102
	19	0.009033	101
	20	0.01077	100
	21	0.01279	99
	22	0.01508	98
	23	0.01767	97
	24	0.02063	96
	25	0.02396	95
	26	0.02768	94
	27	0.03186	93
	28	0.0365	92
	29	0.04163	91
15	30	0.0473	90
	31	0.0535	89
	32	0.06027	88
	33	0.06769	87
	34	0.07571	86
	35	0.08441	85
	36	0.09381	84
	37	0.1039	83
	38	0.1147	82
	39	0.1262	81
	40	0.1384	80
	41	0.1514	79
	42	0.1651	78
	43	0.1796	77
	44	0.1947	76
	45	0.2106	75
	46	0.2271	74
	47	0.2444	73
	48	0.2622	72
	49	0.2807	71
	50	0.2997	70
	51	0.3193	69
	52	0.3394	68
	53	0.3599	67
	54	0.3808	66
	55	0.402	65
	56	0.4235	64
	57	0.4452	63
	58	0.467	62
	59	0.489	61
	60	0.511	60

n	k1	$p(W^+ \le k1)$ $p(W^+ \ge k2)$	k2
	0	0.00001526	136
	1	0.00003052	135
	2	0.00004578	134
	3	0.00007629	133
	4	0.0001068	132
	5	0.0001526	131
	6	0.0002136	130
	7	0.0002899	129
	8	0.0003815	128
	9	0.0005035	127
	10	0.0006561	126
	11	0.0008392	125
	12	0.001068	124
	13	0.001343	123
	14	0.001678	122
	15	0.00209	121
	16	0.002579	120
	17	0.003143	119
	18	0.003815	118
	19	0.004593	117
	20	0.005493	116
	21	0.006546	115
	22	0.007751	114
	23	0.009125	113
	24	0.0107	112
	25	0.01248	111
	26	0.0145	110
	27	0.01677	109
	28	0.01932	108
	29	0.02216	107
	30	0.02533	106
	31	0.02884	105
	32	0.0327	104
	33	0.03696	103
16	34	0.04163	102
	35	0.04672	101
	36	0.05229	100
	37	0.05833	99
	38	0.06487	98
	39	0.07193	97
	40	0.07953	96
	41	0.08768	95
	42	0.09641	94
	43	0.1057	93
	44	0.1156	92
	45	0.1261	91
	46	0.1372	90
	47	0.1489	89
	48	0.1613	88
	49	0.1742	87
	50	0.1877	86
	51	0.2019	85
	52	0.2166	84
	53	0.2319	83
	54	0.2477	82
	55	0.2641	81
	56	0.2809	80
	57	0.2983	79
	58	0.3161	78
	59	0.3343	77
	60	0.3529	76
	61	0.3718	75
	62	0.391	74
	63	0.4104	73
	64	0.4301	72
	65	0.45	71
	66	0.4699	70
	67	0.49	69
	68	0.51	68

3.6 TABLE T6. Table for U_x.

$$p(U_x \leq k) \; for \; n_1 \leq n_2 \; y \; n_1 = 2, 3, ..., 10$$

n2	2	
k/n1	1	2
0	0.3333	0.1667
1	0.6667	0.3333
2		0.6667

n2	3		
k/n1	1	2	3
0	0.25	0.1	0.05
1	0.5	0.2	0.1
2		0.4	0.2
3		0.6	0.35
4			0.5

n2	4			
k/n1	1	2	3	4
0	0.2	0.06667	0.02857	0.01429
1	0.4	0.1333	0.05714	0.02857
2	0.6	0.2667	0.1143	0.05714
3		0.4	0.2	0.1
4		0.6	0.3143	0.1714
5			0.4286	0.2429
6			0.5714	0.3429
7				0.4429
8				0.5571

n2	5				
k/n1	1	2	3	4	5
0	0.1667	0.04762	0.01786	0.007937	0.003968
1	0.3333	0.09524	0.03571	0.01587	0.007937
2	0.5	0.1905	0.07143	0.03175	0.01587
3		0.2857	0.125	0.05556	0.02778
4		0.4286	0.1964	0.09524	0.04762
5		0.5714	0.2857	0.1429	0.0754
6			0.3929	0.2063	0.1111
7			0.5	0.2778	0.1548
8				0.3651	0.2103
9				0.4524	0.2738
10				0.5476	0.3452
11					0.4206
12					0.5

Continuation

n2	6					
k/n1	1	2	3	4	5	6
0	0.1429	0.03571	0.0119	0.004762	0.002165	0.001082
1	0.2857	0.07143	0.02381	0.009524	0.004329	0.002165
2	0.4286	0.1429	0.04762	0.01905	0.008658	0.004329
3	0.5714	0.2143	0.08333	0.03333	0.01515	0.007576
4		0.3214	0.131	0.05714	0.02597	0.01299
5		0.4286	0.1905	0.08571	0.04113	0.02056
6		0.5714	0.2738	0.1286	0.06277	0.03247
7			0.3571	0.1762	0.08874	0.04654
8			0.4524	0.2381	0.1234	0.06602
9			0.5476	0.3048	0.1645	0.08983
10				0.381	0.2143	0.1201
11				0.4571	0.2684	0.1548
12				0.5429	0.3312	0.197
13					0.3961	0.2424
14					0.4654	0.2944
15					0.5346	0.3496
16						0.4091
17						0.4686
18						0.5314

n2	7						
k/n1	1	2	3	4	5	6	7
0	0.125	0.02778	0.008333	0.00303	0.001263	0.0005828	0.0002914
1	0.25	0.05556	0.01667	0.006061	0.002525	0.001166	0.0005828
2	0.375	0.1111	0.03333	0.01212	0.005051	0.002331	0.001166
3	0.5	0.1667	0.05833	0.02121	0.008838	0.004079	0.00204
4		0.25	0.09167	0.03636	0.01515	0.006993	0.003497
5		0.3333	0.1333	0.05455	0.02399	0.01107	0.005536
6		0.4444	0.1917	0.08182	0.03662	0.01748	0.008741
7		0.5556	0.2583	0.1152	0.05303	0.02564	0.01311
8			0.3333	0.1576	0.07449	0.03671	0.01894
9			0.4167	0.2061	0.101	0.0507	0.02652
10			0.5	0.2636	0.1338	0.06876	0.03642
11				0.3242	0.1717	0.09033	0.04866
12				0.3939	0.2159	0.1171	0.0641
13				0.4636	0.2652	0.1474	0.08246
14				0.5364	0.3194	0.183	0.1043
15					0.3775	0.2226	0.1297
16					0.4381	0.2669	0.1588
17					0.5	0.3141	0.1914
18						0.3654	0.2279
19						0.4178	0.2675
20						0.4726	0.31
21						0.5274	0.3552
22							0.4024
23							0.4508
24							0.5

Continuation

n2	8							
k/n1	1	2	3	4	5	6	7	8
0	0.1111	0.02222	0.006061	0.00202	0.000777	0.000333	0.0001554	0.0000777
1	0.2222	0.04444	0.01212	0.00404	0.001554	0.000666	0.0003108	0.0001554
2	0.3333	0.08889	0.02424	0.008081	0.003108	0.001332	0.0006216	0.0003108
3	0.4444	0.1333	0.04242	0.01414	0.005439	0.002331	0.001088	0.0005439
4	0.5556	0.2	0.06667	0.02424	0.009324	0.003996	0.001865	0.0009324
5		0.2667	0.09697	0.03636	0.01476	0.006327	0.002953	0.001476
6		0.3556	0.1394	0.05455	0.02253	0.00999	0.004662	0.002331
7		0.4444	0.1879	0.07677	0.03263	0.01465	0.006993	0.003497
8		0.5556	0.2485	0.1071	0.04662	0.02131	0.01026	0.005206
9			0.3152	0.1414	0.06371	0.02964	0.01445	0.007382
10			0.3879	0.1838	0.08547	0.04063	0.02005	0.01033
11			0.4606	0.2303	0.1111	0.05395	0.02704	0.01406
12			0.5394	0.2848	0.1422	0.07093	0.03605	0.01896
13				0.3414	0.1772	0.09058	0.04693	0.02494
14				0.404	0.2176	0.1142	0.0603	0.03248
15				0.4667	0.2618	0.1412	0.07599	0.04149
16				0.5333	0.3108	0.1725	0.09464	0.05245
17					0.3621	0.2068	0.1159	0.06519
18					0.4165	0.2454	0.1405	0.08026
19					0.4716	0.2864	0.1678	0.09744
20					0.5284	0.331	0.1984	0.1172
21						0.3773	0.2317	0.1393
22						0.4259	0.2679	0.1641
23						0.4749	0.3063	0.1911
24						0.5251	0.3472	0.2209
25							0.3894	0.2527
26							0.4333	0.2869
27							0.4775	0.3227
28							0.5225	0.3605
29								0.3992
30								0.4392
31								0.4796
32								0.5204

Continuation

n2	9								
k/n1	1	2	3	4	5	6	7	8	9
0	0.1	0.01818	0.004545	0.001399	0.0004995	0.0001998	0.00008741	0.00004114	0.00002057
1	0.2	0.03636	0.009091	0.002797	0.000999	0.0003996	0.0001748	0.00008227	0.00004114
2	0.3	0.07273	0.01818	0.005594	0.001998	0.0007992	0.0003497	0.0001645	0.00008227
3	0.4	0.1091	0.03182	0.00979	0.003497	0.001399	0.0006119	0.0002879	0.000144
4	0.5	0.1636	0.05	0.01678	0.005994	0.002398	0.001049	0.0004936	0.0002468
5		0.2182	0.07273	0.02517	0.009491	0.003796	0.001661	0.0007816	0.0003908
6		0.2909	0.1045	0.03776	0.01449	0.005994	0.002622	0.001234	0.000617
7		0.3636	0.1409	0.05315	0.02098	0.008791	0.003934	0.001851	0.0009255
8		0.4545	0.1864	0.07413	0.02997	0.01279	0.005769	0.002756	0.001378
9		0.5455	0.2409	0.0993	0.04146	0.01798	0.008217	0.003949	0.001995
10			0.3	0.1301	0.05594	0.02478	0.01145	0.005553	0.002818
11			0.3636	0.165	0.07343	0.03317	0.01556	0.00761	0.003887
12			0.4318	0.207	0.09491	0.04396	0.02089	0.01032	0.005306
13			0.5	0.2517	0.1199	0.05674	0.02745	0.0137	0.007096
14				0.3021	0.1489	0.07233	0.03558	0.01798	0.009379
15				0.3552	0.1818	0.09051	0.04537	0.0232	0.01222
16				0.4126	0.2188	0.1119	0.05708	0.02962	0.01573
17				0.4699	0.2592	0.1361	0.0708	0.03723	0.01999
18				0.5301	0.3032	0.1638	0.08689	0.04636	0.02515
19					0.3497	0.1942	0.1052	0.05697	0.03126
20					0.3986	0.228	0.1261	0.0694	0.0385
21					0.4491	0.2643	0.1496	0.08359	0.04696
22					0.5	0.3035	0.1755	0.09979	0.05675
23						0.3445	0.2039	0.1179	0.06796
24						0.3878	0.2349	0.1383	0.08075
25						0.432	0.268	0.1606	0.09513
26						0.4773	0.3032	0.1852	0.1112
27						0.5227	0.3403	0.2117	0.129
28							0.3788	0.2404	0.1487
29							0.4185	0.2707	0.1701
30							0.4591	0.3029	0.1933
31							0.5	0.3365	0.2181
32								0.3715	0.2447
33								0.4074	0.2729
34								0.4442	0.3024
35								0.4813	0.3332
36								0.5187	0.3652
37									0.3981
38									0.4317
39									0.4657
40									0.5

Continuation

n2	10									
k/n1	1	2	3	4	5	6	7	8	9	10
0	0.09091	0.01515	0.003497	0.000999	0.000333	0.0001249	0.00005142	0.00002285	0.00001083	5.413×10^{-6}
1	0.1818	0.0303	0.006993	0.001998	0.000666	0.0002498	0.0001028	0.00004571	0.00002165	0.00001083
2	0.2727	0.06061	0.01399	0.003996	0.001332	0.0004995	0.0002057	0.00009141	0.0000433	0.00002165
3	0.3636	0.09091	0.02448	0.006993	0.002331	0.0008741	0.0003599	0.00016	0.00007578	0.00003789
4	0.4545	0.1364	0.03846	0.01199	0.003996	0.001499	0.000617	0.0002742	0.0001299	0.00006495
5	0.5455	0.1818	0.05594	0.01798	0.006327	0.002373	0.000977	0.0004342	0.0002057	0.0001028
6		0.2424	0.08042	0.02697	0.009657	0.003746	0.001543	0.0006856	0.0003248	0.0001624
7		0.303	0.1084	0.03796	0.01399	0.005495	0.002314	0.001028	0.0004871	0.0002436
8		0.3788	0.1434	0.05295	0.01998	0.007992	0.003394	0.001531	0.0007253	0.0003626
9		0.4545	0.1853	0.07093	0.02764	0.01124	0.004833	0.002194	0.00105	0.000525
10		0.5455	0.2343	0.09391	0.03763	0.01561	0.006787	0.003108	0.001494	0.0007523
11			0.2867	0.1199	0.04962	0.02098	0.009255	0.004274	0.002068	0.001045
12			0.3462	0.1518	0.0646	0.02797	0.01249	0.005828	0.002836	0.00144
13			0.4056	0.1868	0.08225	0.03634	0.01651	0.00777	0.00381	0.001943
14			0.4685	0.2268	0.1032	0.0467	0.02154	0.01026	0.005066	0.002598
15			0.5315	0.2697	0.1272	0.05894	0.02766	0.01332	0.006636	0.003421
16				0.3177	0.1548	0.07355	0.03512	0.01714	0.008606	0.004465
17				0.3666	0.1855	0.09028	0.04391	0.02171	0.01101	0.005748
18				0.4196	0.2198	0.1099	0.0544	0.02726	0.01396	0.007345
19				0.4725	0.2567	0.1317	0.06654	0.0338	0.01749	0.009272
20				0.5275	0.297	0.1566	0.08063	0.04157	0.02174	0.01162
21					0.3393	0.1838	0.09662	0.05055	0.02674	0.0144
22					0.3839	0.2139	0.1148	0.06099	0.03263	0.01773
23					0.4296	0.2461	0.1349	0.07286	0.03945	0.02163
24					0.4765	0.2811	0.1574	0.08641	0.04736	0.02621
25					0.5235	0.3177	0.1819	0.1015	0.05638	0.03151
26						0.3564	0.2087	0.1185	0.06665	0.03763
27						0.3962	0.2374	0.1371	0.0782	0.0446
28						0.4374	0.2681	0.1577	0.09116	0.05256
29						0.4789	0.3004	0.18	0.1055	0.0615
30						0.5211	0.3345	0.2041	0.1214	0.07157
31							0.3698	0.2299	0.1388	0.08275
32							0.4063	0.2574	0.1577	0.09516
33							0.4434	0.2863	0.1781	0.1088
34							0.4811	0.3167	0.2001	0.1237
35							0.5189	0.3482	0.2235	0.1399
36								0.3809	0.2483	0.1575
37								0.4143	0.2745	0.1763
38								0.4484	0.3019	0.1965
39								0.4827	0.3304	0.2179
40								0.5173	0.3598	0.2406
41									0.3901	0.2644
42									0.4211	0.2894
43									0.4524	0.3153
44									0.4841	0.3421
45									0.5159	0.3697
46										0.398
47										0.4267
48										0.4559
49										0.4853
50										0.5147

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